3D Computer Vision
Syllabus

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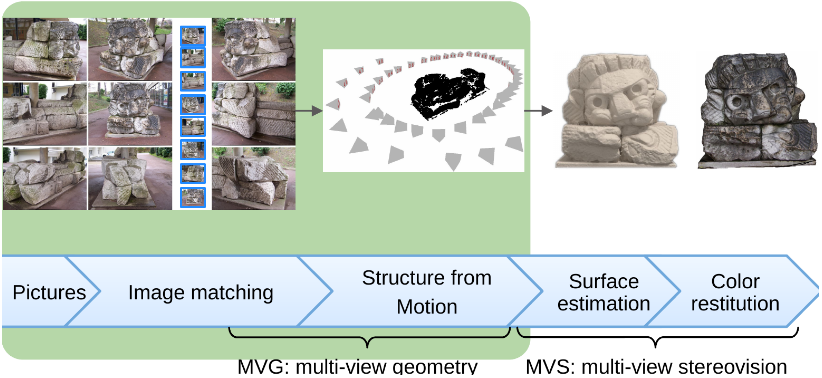
Structure from Motion

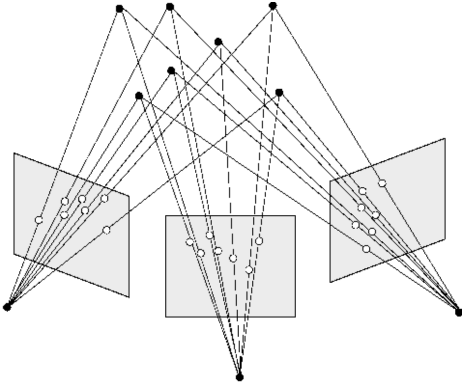


We will cover the first part of this Stereo pipeline, MVG, and first part of MVS but only with two views.

- Output of MVG: point cloud and camera positions and orientations
- Output of MVS: a mesh surface with colors

Bundle Adjustment

- Observed points in images are central projections of unknown 3D points.
- Each image provides a bundle of half-lines containing the 3D points.
- We must orient the bundles so that the lines intersect.



Source: Manolis Lourakis

One equation to rule them all

$$\arg \min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij}\; d\left(\mathbf{x}_{ij}, \Pi_i(R_i \mathbf{X}_j + T_i)\right)^2 .$$

- $\mathbf{x}_{ij}$: point number j (among n) in view i (among m), pixel coordinates.
- $\epsilon_{ij} \in \{0, 1\}$: visibility of point j in view i.
- d: Euclidean distance in image.
- Π_i: 2D projection operator in view i from 3D.

Unknowns:

- $\mathbf{X}_j$: 3D point in a fixed coordinate frame.
- R_i: rotation matrix of coordinate frame wrt view i.
- T_i: position of origin O of coordinate frame wrt view i.

Session 1

$$\arg \min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij} \, d\left(\mathbf{x}_{ij}, \Pi_i(R_i \mathbf{X}_j + T_i)\right)^2 .$$

- Expression of projection model Π_i (i.e., camera model)
- Case $m = 2$, $T_L = T_R = 0$ or all X_j on a single plane:

  $$\forall i \in \{L, R\}, \forall j, \; d\left(\mathbf{x}_{ij}, \Pi_i(R_i \mathbf{X}_j + 0)\right) = 0.$$

  Then $\mathbf{x}_{Rj} = H_{\Pi_R, R_R}(\mathbf{x}_{Lj})$, and the *homography* H can be determined from four pairs $\{\mathbf{x}_{Lj}, \mathbf{x}_{Rj}\}$. Application: panorama.
- Case $m = 1$, $n \geq 6$ pairs $(\mathbf{x}_j, \mathbf{X}_j)$, recover all info about the camera (camera calibration from 3D rig):

  $$\arg \min_{\Pi, R, T} \sum_{j} d\left(\mathbf{x}_{ij}, \Pi(R \mathbf{X}_j + T)\right)^2 .$$

- Case $m \geq 3$, single camera Π, 4 (or more) pairs $(\mathbf{x}_j, \mathbf{X}_j)$, all X_j on a common plane (camera calibration from 2D rig):

  $$\arg \min_{\Pi, \{R_i\}, \{T_i\}} \sum_{ij} d\left(\mathbf{x}_{ij}, \Pi(R_i \mathbf{X}_j + T_i)\right)^2 .$$

Session 2

$$\arg\min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij}\, d\left(\mathbf{x}_{ij}, \Pi_i(R_i\mathbf{X}_j + T_i)\right)^2 .$$

Case $m = 2$ views.

- From $n \geq 5$ pairs $\{\mathbf{x}_{Lj}, \mathbf{x}_{Rj}\}$ and known Π_L, Π_R, deduce a bilinear constraint $\forall j$, $E_{R_R,T_R}(\mathbf{x}_{Lj}, \mathbf{x}_{Rj}) = 0$ (essential matrix). Recover (R_R, T_R) from E.

$$\arg\min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i\in\{L,R\},j} \epsilon_{ij}\, d\left(\mathbf{x}_{ij}, \Pi_i(R_i\mathbf{X}_j + T_i)\right)^2 .$$

- From $n \geq 7$ pairs $\{\mathbf{x}_{Lj}, \mathbf{x}_{Rj}\}$ and *unknown* Π_L, Π_R, deduce a bilinear constraint $\forall j$, $F(\mathbf{x}_{Lj}, \mathbf{x}_{Rj}) = 0$ (fundamental matrix).
- From n pairs $\{\mathbf{x}_{Lj}, \mathbf{x}_{Rj}\}$, maximize the number of matching pairs up to tolerance $\tau \geq 0$ (RANSAC algorithm)

$$\max \sum_{i\in\{L,R\},j} \epsilon_{ij}$$

$$\text{s.t. } \exists \Pi_L, \Pi_R, \{R_i\}, \{T_i\}, \{X_j\},\ d\left(\mathbf{x}_{ij}, \Pi_i(R_i\mathbf{X}_j + T_i)\right) \leq \tau.$$

Sessions 3&4

$$\arg \min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij} \; d\left(\mathbf{x}_{ij}, \Pi_i(R_i \mathbf{X}_j + T_i)\right)^2 .$$

Case $m = 2$ views, $R_L = R_R = I$, $T_L = 0$, $T_R = Be_1$ (rectified pair).

- Go from generic poses to rectified pair with fundamental matrix F.
- For all $\mathbf{x}_{Lj}$, $j \in \{\text{pixels of left image}\}$, find corresponding pixel $\mathbf{x}_{Rj}$, hence $\mathbf{X}_j$ by triangulation.
- Two categories of methods: local/global.

Session 5

$$\arg \min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij} \; d\left(\mathbf{x}_{ij}, \Pi_i(R_i \mathbf{X}_j + T_i)\right)^2 .$$

Case $m \geq 3$, multi-view stereovision.

- For $m = 3$, trilinear constraints $\mathcal{T}(\mathbf{x}_{1j}, \mathbf{x}_{2j}, \mathbf{x}_{3j}) = 0$. Recovery of tensor $\mathcal{T}$.
- Recover R_{m+1} and T_{m+1} from known pairs $(\mathbf{x}_{m+1j}, \mathbf{X}_j)$ (resection, PnP Perspective from n Points):

$$\arg \min_{R_{m+1}, T_{m+1}} \sum_{j} \epsilon_{m+1j} \; d\left(\mathbf{x}_{ij}, \Pi_{m+1}(R_{m+1} \mathbf{X}_j + T_{m+1})\right)^2 .$$

$\Rightarrow$ incremental multi-view pipeline.

- Special case $T_i = B_i v$ with v a fixed vector and known B_i:

$$\arg \min_{\{R_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij} \; d\left(\mathbf{x}_{ij}, \Pi_i(R_i \mathbf{X}_j + B_i v)\right)^2 .$$

$\Rightarrow$ Light field imagery, epipolar plane imagery

Session 6

$$\arg \min_{\{R_i\},\{T_i\},\{\mathbf{X}_j\}} \sum_{i,j} \epsilon_{ij} \; d\left(\mathbf{x}_{ij}, \Pi_i(R_i\mathbf{X}_j + T_i)\right)^2 .$$

Case $m = 2$ views, find interest points $\{\mathbf{p}_{Lk}\}$ and $\{\mathbf{p}_{Rk'}\}$. Deduce pairs $j = (k, k')$ of matching points $(\mathbf{x}_{Lj}, \mathbf{x}_{Rj})$ with $\mathbf{x}_{Lj} = \mathbf{p}_{Lk}$ and $\mathbf{x}_{Rj} = \mathbf{p}_{Rk'}$.