A LEVEL SET METHOD FOR THE INVERSE EEG/MEG PROBLEM

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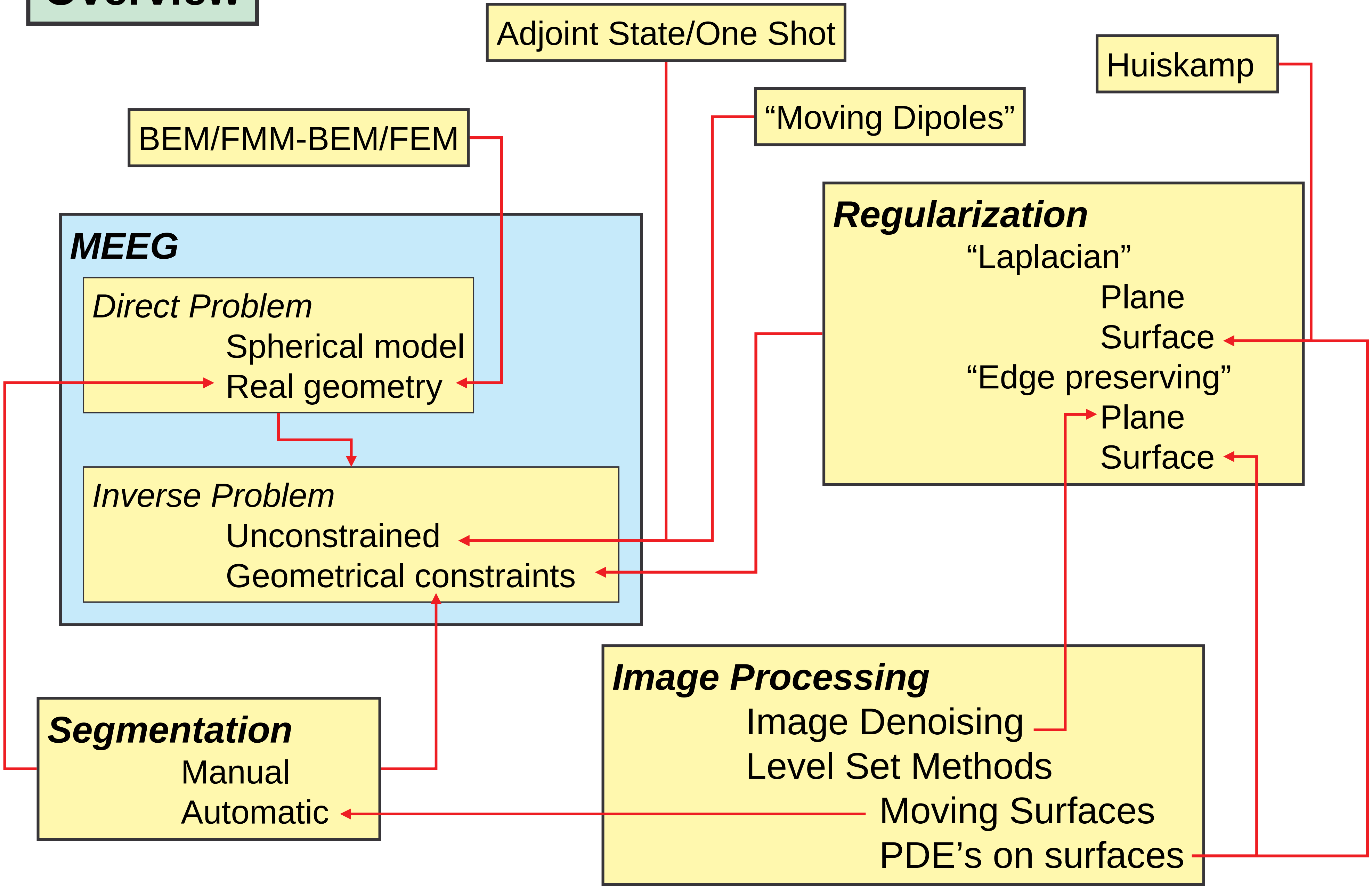
Overview


Adjoint State/One Shot
Huiskamp
"Moving Dipoles"
BEM/FMM-BEM/FEM
MEEG
Direct Problem
Spherical model
Real geometry
Inverse Problem
Unconstrained
Geometrical constraints
Regularization
"Laplacian"
Plane
Surface
"Edge preserving"
Plane
Surface
Segmentation
Manual
Automatic
Image Processing
Image Denoising
Level Set Methods
Moving Surfaces
PDE's on surfaces

Direct problem (I)

Brain activity → Direct problem → Electric/Magnetic fields

Electric/Magnetic fields → Inverse problem → Brain activity

Direct problem: $\mathbf{J}^p \rightarrow V, \mathbf{B}$

$$\nabla.(\sigma \nabla V) = \nabla.\mathbf{J}^p$$

$$\mathrm{B}(\mathbf{r}) = \mathrm{B}_0(\mathbf{r}) + \frac{\mu_0}{4\pi}\int_\Omega V(\mathbf{r}')\nabla'\sigma \times \nabla'(\frac{1}{|\mathbf{r}-\mathbf{r}'|})d\mathbf{r}'$$

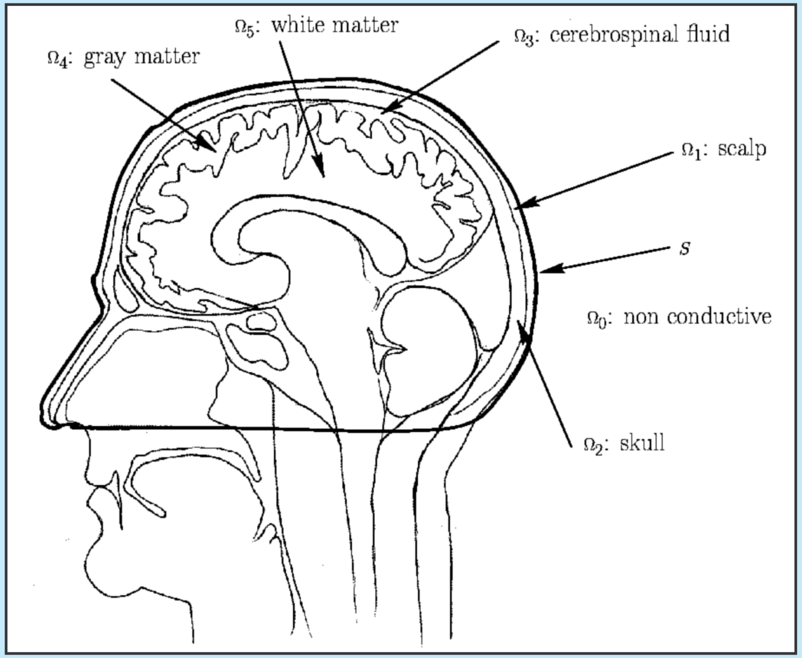
$$\mathrm{B}_0(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_\Omega \mathbf{J}^p(\mathbf{r}') \times \nabla'(\frac{1}{|\mathbf{r}-\mathbf{r}'|})d\mathbf{r}'$$

[Cohen 68, Geselowitz 67, Hamalainen/Ilmoniemi 84, Mosher/Leahy 97]

Direct Problem (II)

Equivalent Geometry

- **Exact Solutions** (spheres)
  [de Munck 92, Berg/Sherg 94, Zhang 95]



Exact Geometry

- **Finite Differences** *[Hedoux 95]*

- **Finite Elements** (FEM)
  [Dale/Sereno 93, Baillet/Garnero 98, Clerc/Dervieux/Faugeras/Keriven/Kybic/Papadopoulo 02]

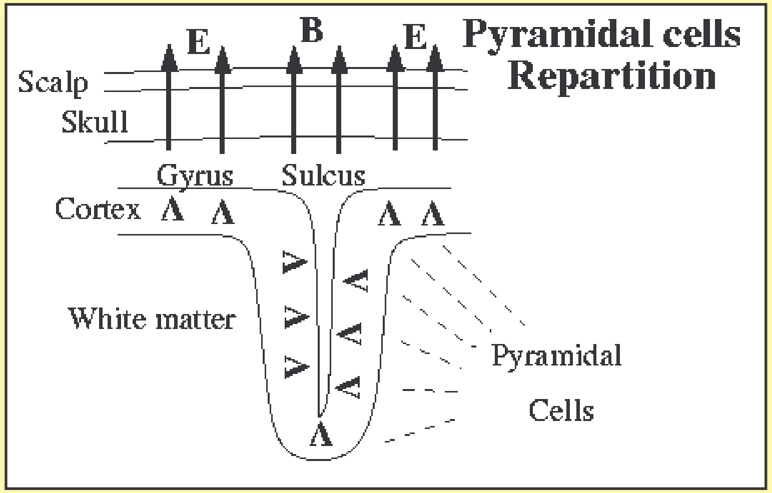
- **Integral Methods**
  (BEM) *[Sarvas 87, Ferguson/Stroink 97]*
  (FMM-BEM) [Clerc/Faugeras/Keriven/Kybic/Papadopoulo 02]

Inverse Problem (I)

$$V, \mathbf{B} \rightarrow \mathbf{J}^p$$

- **Moving Current Dipoles**
  [Mosher/Lewis/Leahy 92, Mosher/Leahy 99]
- **Geometrical constraints**
  [Baillet/Garnero 97, Faugeras et al 99]

- **Here, $\mathbf{J}^p$ is a density of current dipoles on the cortical surface, and normal to it.**



Inverse Problem (II)

$V, \mathbf{B}$ **are now linear functions of** $\mathbf{J}^p$

Regularization:

$$\inf_{\mathbf{J}^p} \quad \frac{1}{2}\sum_{i=1}^{n} ||V_i(\mathbf{J}^p) - V_i^{mes}||^2 + \frac{1}{2}\sum_{i=1}^{m} ||\mathbf{B}_i(\mathbf{J}^p) - \mathbf{B}_i^{mes}||^2 + \alpha \int_S |\nabla \mathbf{J}^p|^2 dS$$

Image Denoising …

… is an inverse problem!

- Direct problem: $I \longrightarrow I_N$
- Inverse problem: $I_N \longrightarrow I$

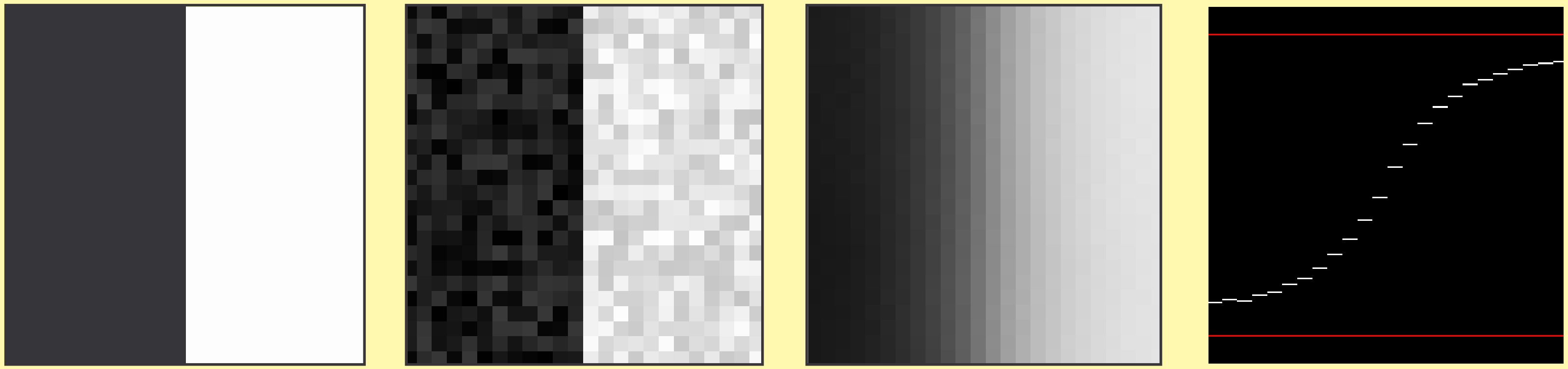
$$\inf_I \frac{1}{2}\int_\Omega (I - I_N)^2 dx + \alpha \int_\Omega g(|\nabla I|) dx$$

"Data term" — Regularization term

Isotropic Diffusion

$$g(s) = \frac{1}{2}s^2$$ *[Tikhonov 63, Witkin 83,Koenderink 84]*

Gradient steepest descent: $$\frac{\partial I}{\partial t} = (I_N - I) + \alpha \Delta I$$



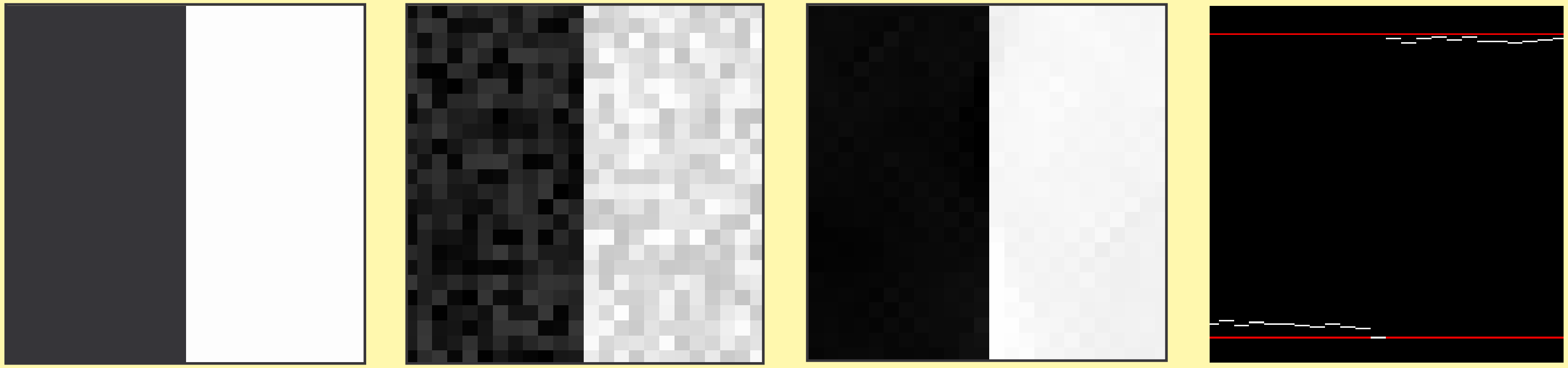
Original Noisy Denoised Horiz. cut

Not “edge preserving”!

Anisotropic Diffusion

Total variation $g(s) = s$ *[Rudin/Osher/Fatemi 92]*

Gradient steepest descent: $$\frac{\partial I}{\partial t} = (I_N - I) + \alpha div(\frac{\nabla I}{|\nabla I|})$$



Original Noisy Denoised Horiz. cut

Edge preserving!

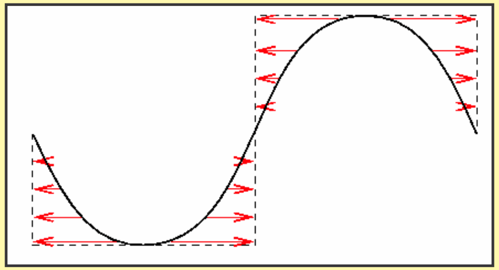
Other possible image processing tools

Other anisotropic diffusions

• ***[Perona/Malik 90, Malladi/Sethian 96, Alvarez/Mazora 94, ….]***

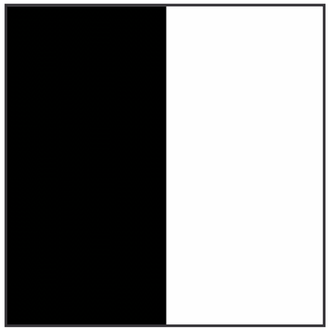
Reaction (shock filters)

• ***[Osher/Rudin 90, Kornprobst/Derich/Aubert 96]***

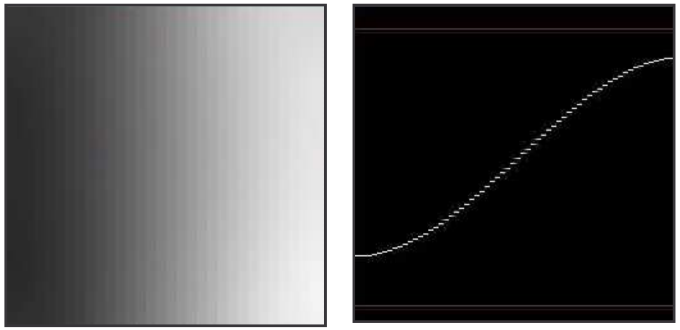


MEEG – Constrained on a plane

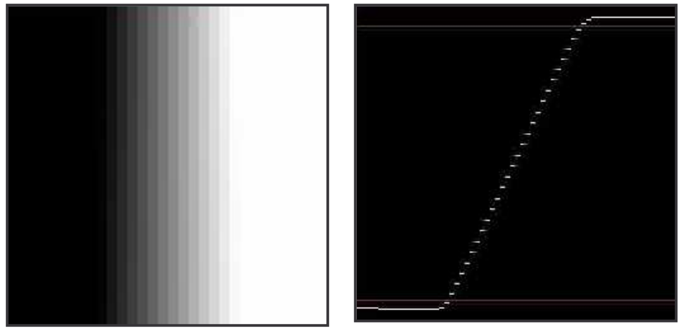
EEG only, 16 electrodes



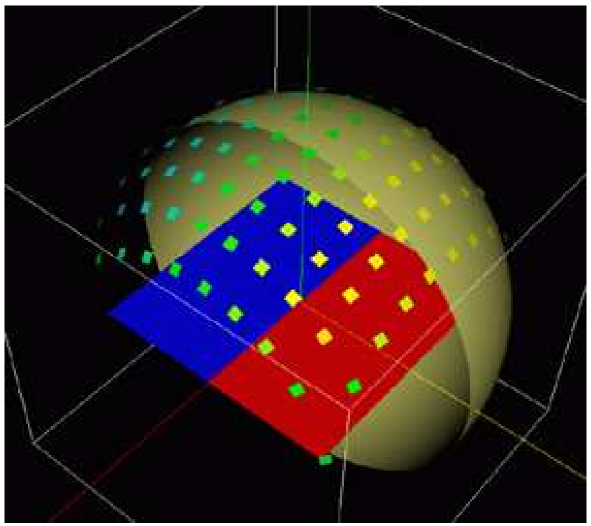
Activity



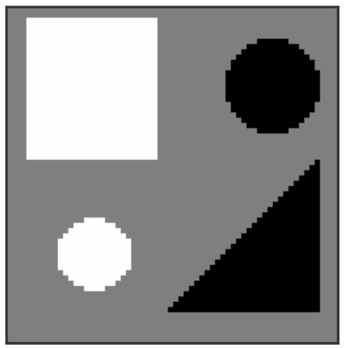
Recovered (isotrop. diff.)



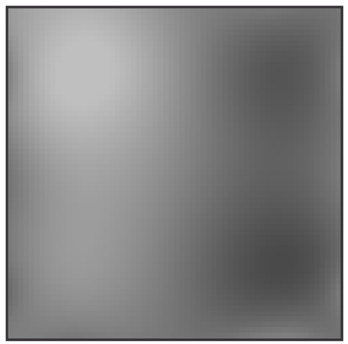
Recovered (Total variation)



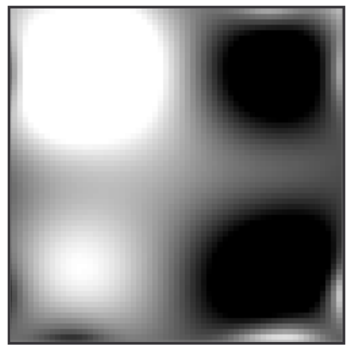
Configuration with 100 electrodes



Activity

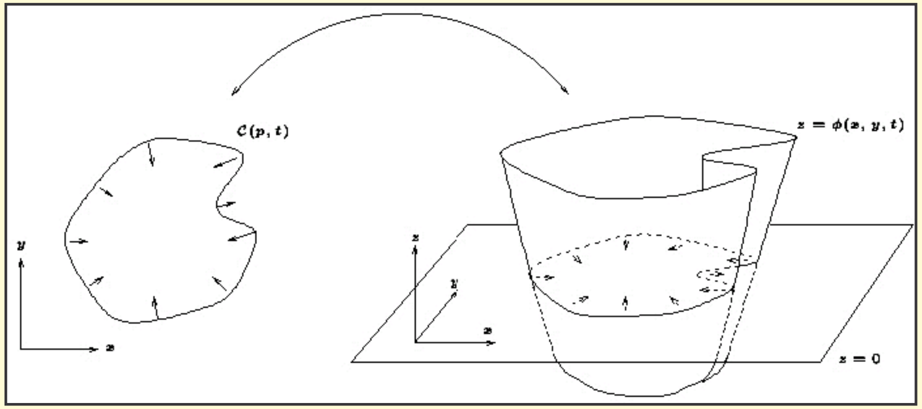


Isotropic



Total variation

Level Set (I): Cortical surface extraction

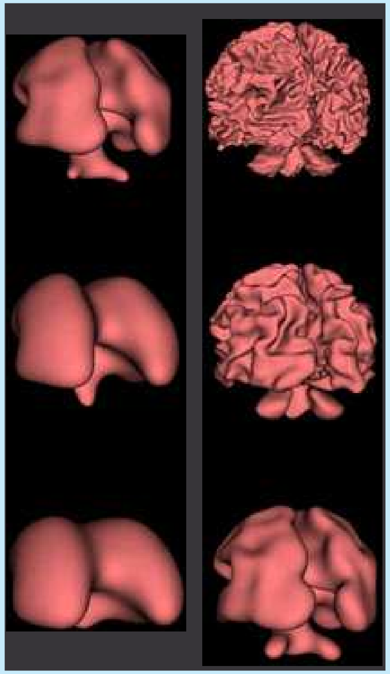


Level Set Method *[Osher/Sethian 88]*

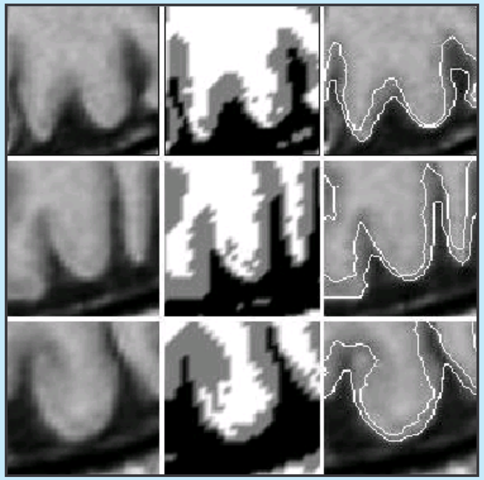
$$\frac{\partial C}{\partial t} = \beta \mathbf{N}$$

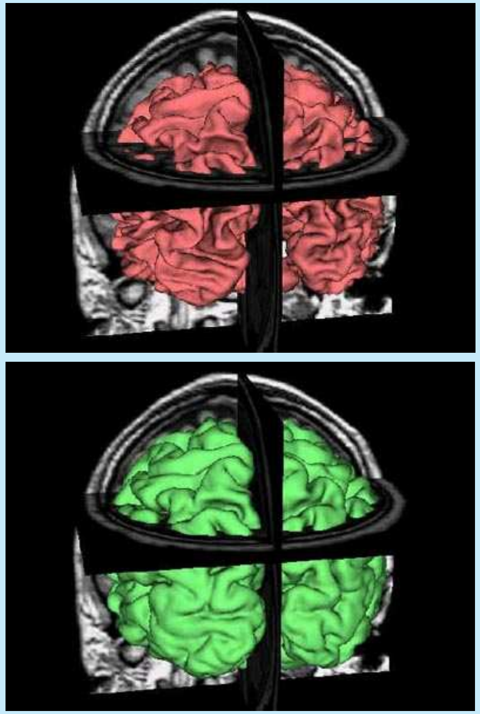
$$\Updownarrow \quad C(.,t) = \{\phi(x,t) = 0\}$$

$$\frac{\partial \phi}{\partial t} = \beta |\nabla \phi|$$



Application to cortex segmentation
from MRI *[Faugeras/Gomes 99]*
(Two coupled surfaces)





Level Set (II): PDE's on surfaces

[Bertalmio/Cheng/Osher/Sapiro 00]

- **Surface** $S = \{x \in \mathbf{R}^3 : \phi(x) = 0\}$
- **Surface PDE** $\frac{\partial u}{\partial t} = \Delta_S u$
- **Extension** $\nabla u . \nabla \phi = 0$
- **3D PDE** $\frac{\partial u}{\partial t} = \frac{1}{|\nabla \phi|} \nabla . (P_{\nabla \phi} \nabla u |\nabla \phi|)$

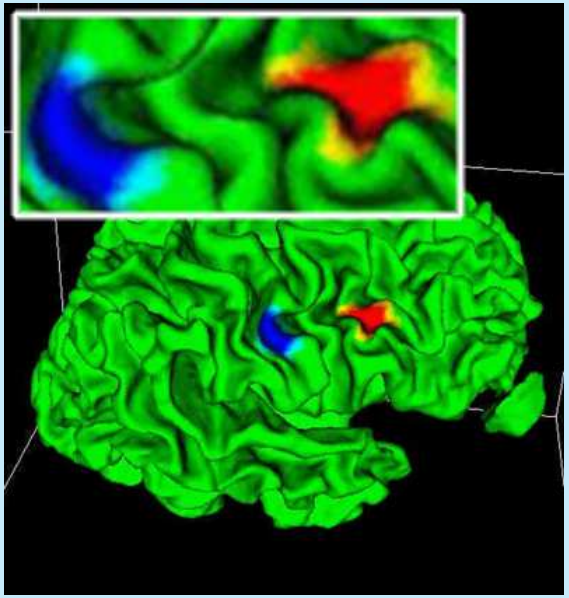
(projection onto the plane normal to a given vector)

- **Simpler and more accurate than finite difference formula on triangulated Surfaces ([Huiskamp])**
- **Allows other PDE's (e.g. total variation)**

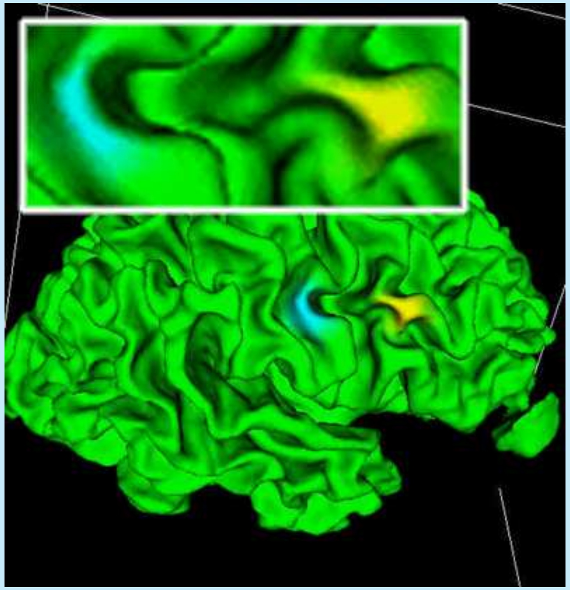
MEEG – Constrained to the cortex

$$\inf_{\mathbf{J}^p} \text{"Data term"} + \alpha \int_S g(|\nabla_S \mathbf{J}^p|)\, dS$$

Real geometry, Synthetic data + noise, EEGx64 + MEGx64



Activity

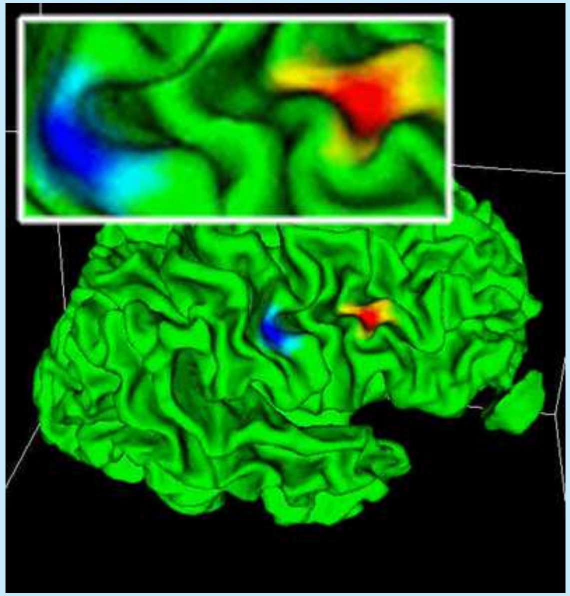


Isotropic diffusion

regul.: $\int |\nabla \mathbf{J}^p|^2$

$$\frac{\partial u}{\partial t} = \nabla.(P_{\nabla\phi} \nabla u)$$

$$\phi(x) = d(x, S),\ \mathbf{J}^p = u|_S$$



Total variation

regul.: $\int |\nabla \mathbf{J}^p|$

$$\frac{\partial u}{\partial t} = \nabla.(\frac{P_{\nabla\phi} \nabla u}{|P_{\nabla\phi} \nabla u|})$$

$$\phi(x) = d(x, S),\ \mathbf{J}^p = u|_S$$

Conclusion

- Novel framework
  - Simple and accurate surface Laplacian computation
  - Allowing other regularization terms (total variation, …) **preserving edges**

Future work

- Other regularization and image processing tools (shocks?)
- Which functional space for $\mathbf{J}^p$?
- Adding time (space-time regularization preserving discontinuities)
- Real data