Establishing Local Correspondences towards Compact Representations of Anatomical Structures*

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Abstract. Computer-aided diagnosis is often based on comparing a structure of interest with prior models. Such comparison requires automatic techniques in determining prior models from a set of examples and establishing local correspondences between the structure and the model. In this paper we propose a variational technique for solving the correspondence problem. The proposed method integrates a powerful representation for shapes (implicit representations), a state-of-the art criterion for global registration (mutual information) and an efficient technique to recover local correspondences (free form deformations) that guarantees one-to-one mapping. The proposed registration paradigm can register open/close structures of arbitrary dimension. Local correspondences can then be used to build compact representations for a structure of interest according to a set of training examples. The registration and statistical modeling of Systolic Left Ventricle in Ultrasonic images demonstrate the potential of the proposed technique.

1 Introduction

Organ modeling is a critical component of medical image analysis. To this end, one would like to recover a compact representation that can capture the variation of the structure of interest across individuals. Building such representations requires establishing correspondences across the set of training examples. Such objective can be either based in a pure geometric space (shape) or in the visual space (intensity properties) or in a joint space. Quite often, correspondences are user-determined, which is a non-efficient and time consuming process. Once correspondences have been established, the modeling can take place according to various statistical methods leading to compact representations that can be used for detection, segmentation, tracking, etc. of structures of interest.

Shape/Image registration [7] is an evolving research activity in medical imaging [11]. One can define the registration problem as follows: recover a transformation between a source and a target shape that results in meaningful correspondences between their basic elements. Such definition involves three aspects.

The selection of an appropriate representation for the structures of interest. Cloud of points, parametric (concrete) structures (e.g. B-splines), compact representations (e.g. medial axis), etc. are often considered as a feature space when solving the registration

* Due to the lack of space, a significant amount of prior literature related with the registration problem is omitted and only a small portion of results is shown.

problem. Compact representations are appropriate when seeking global registration and quite in-efficient when seeking for local correspondences. Concrete structures are considered to be a good option for local registration. Their limitations are related with the selection of the representation, the number of basic components and the ability to describe open and multi-component structures. Space coordinates (cloud of points) are simple but rather inconsistent representations that do not provide sufficient support in estimating the local geometric characteristics.

The transformation can be either global or local. Global registration techniques aim to recover a linear motion model that is applicable to the entire structure of interest. Such techniques are a compromise between robustness, low complexity and acceptable registration performance. Medical image analysis is a domain where accurate correspondences are required. Local registration techniques focus on recovering an element-based flow that creates correspondences in the pixel level. These techniques aim to solve an ill-posed problems since the number of constraints is inferior to the number of unknown variables.

Last, but not least, given a feature space and a selection of the form of transformation one should define an appropriate mathematical framework to recover the optimal registration parameters. Stochastic, variational, graph-based optimization functions are examples with known strengths and limitations.

In this paper we propose an hierarchical registration method for establishing local correspondences that can deal with the ill-poseness of the local registration problem. We represent the structures of interest (shapes) in a higher dimension using an implicit representation that is derived from the powerful space of distance transforms. Global registration for an arbitrary motion model is obtained using the mutual information criterion [2, 12]. Local deformations are considered to be a complementary (to the global motion model) registration field. In order to preserve one-to-one correspondence, a slight variance of the B-Spline based free form deformation model (FFD) [10] is used to perform local registration, by incrementally deforming a control lattice overlaid on the structure of interest. The task of generating a compact representation using a principal component analysis technique [3] from a set of training examples is considered to demonstrate the potentials of the proposed registration algorithm.

2 Distance Transforms, Mutual Information & Rigid Registration

The definition of the feature space is a critical component of the registration process. The use of point clouds [1], deformable models [13], fourier descriptors are some alternatives. Such representations are powerful enough to capture a certain number of local deformations. However, one can claim that a significant number of components is required to cope with important shape deformations in these representations, and their extension to describe structures of higher dimension than curves and surfaces is in most of the cases not trivial. More advanced techniques are based on implicit geometric characteristics of a structure of interest, like the curvature, medial axes, normals , etc. or combination of them. The estimation of such implicit properties is a difficult task that often requires the parameterization of the structure.

We consider an implicit representation for the source and the target structures [8]. Euclidean distance transforms are a powerful selection to embed a structure of interest into a higher dimension. Let $\Phi : \Omega \rightarrow R^+$ be a function that refers to a distance

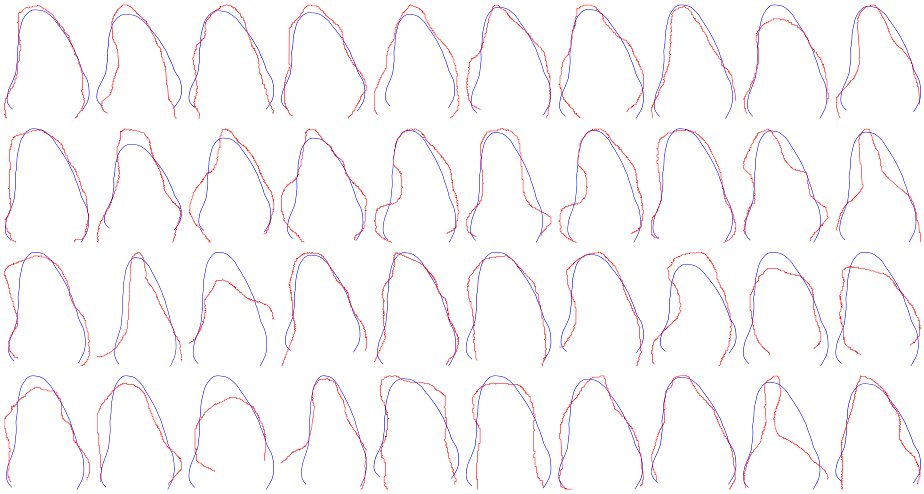

Fig. 1. Rigid Registration for User-Determined Ground Truth (Systole) from the Left Ventricle of Ultrasonic Images (multiple views). (blue) target mean shape, (red) registered source shape.

transform representation for a given shape $\mathcal{S}$:

$$\Phi_{\mathcal{S}}(x,y) = \begin{cases} 0, & (x,y) \in \mathcal{S} \\ D((x,y),\mathcal{S}), & (x,y) \in \Omega - \mathcal{S} \end{cases}$$

where $D((x,y),\mathcal{S})$ refers to the min distance between a pixel (x,y) in the embedding space and the shape $\mathcal{S}$. The signed distance function is a more powerful representation and can be used to describe close structures. The use of implicit representations provides additional support to the registration process since one would like to align the original structures as well as their clones that are positioned coherently in the image/volume plane.

The selected shape representation is translation/rotation invariant. Scale variations can be considered as a global illumination change in the space of distance transforms. Therefore, registration under scale variations is equivalent with matching different modalities that refer to the same structure of interest. The information theoretic criterion, Mutual Information, can address such matching objective, since Mutual information is an invariant technique according to a monotonic transformation of the two input random variables. Such criterion is based on the global characteristics of the structures of interest. In order to facilitate the notation let us denote: **(i)** the source representation $\Phi_{\mathcal{D}}$ as f, and **(ii)** the target representation $\Phi_{\mathcal{S}}$ as g. In the most general case, registration is equivalent with recovering the parameters $\mathbf{\Theta} = (\theta_1, \theta_2, ..., \theta_N)$ of a parametric transformation A such that the mutual information between $f_\Omega = f(\Omega)$ and $g_\Omega^A = g\big(A(\mathbf{\Theta}; \Omega)\big)$ is maximized for a given sample domain Ω;

$$MI(X^{f_\Omega}, X^{g_\Omega^A}) = \mathcal{H}\left[X^{f_\Omega}\right] + \mathcal{H}\left[X^{g_\Omega^A}\right] - \mathcal{H}\left[X^{f_\Omega, g_\Omega^A}\right]$$

where $\mathcal{H}$ represents the differential entropy. Such quantity represents a measure of uncertainty, variability or complexity and consists of three components: (i) the entropy of the model, (ii) the entropy of the projection of the model given the transformation, and (iii) the joint entropy between the model and the projection that encourages transformations where f explains g. One can use the above criterion and an arbitrary transformation (rigid, affine, homographic, quadratic) to perform global registration that is equivalent with minimizing:

$$E(A(\mathbf{\Theta})) = -MI(X^{f_\Omega}, X^{g^A_\Omega}) = -\iint_{\mathcal{R}^2} p^{f_\Omega, g^A_\Omega}(l_1, l_2) log \frac{p^{f_\Omega, g^A_\Omega}(l_1, l_2)}{p^{f_\Omega}(l_1) p^{g^A_\Omega}(l_2)} dl_1 dl_2$$

where (i) p^{f_Ω} corresponds to the probability density in $f_\Omega \left([\Phi_\mathcal{D}(\Omega)] \right)$, (ii) $p^{g^A_\Omega}$ corresponds to density in $g^A_\Omega \left([\Phi_\mathcal{S}(A(\mathbf{\Theta}; \Omega))] \right)$, and (iii) p^{f_Ω, g^A_Ω} is the joint density. Such framework can account for various global motion models. Towards a continuous form of the criterion, a non-parametric Gaussian Kernel density model can be considered to approximate the joint density, leading to the following expression:

$$p^{f_\Omega, g^A_\Omega}(l_1, l_2) = \frac{1}{V(\Omega)} \iint_\Omega G(l_1 - f(\mathbf{x}), l_2 - g(A(\mathbf{\Theta}; \mathbf{x}))) d\mathbf{x}$$

where $[G(l_1 - f(\mathbf{x}), l_2 - g(A(\mathbf{\Theta}; \mathbf{x})))]$ represents a two dimensional zero-mean differentiable Gaussian kernel. A similar approach can be considered in defining $p^{f_\Omega}(l_1)$ and $p^{g^A_\Omega}(l_2)$ using a 1D Gaussian kernel. The calculus of variations with a gradient descent method [6] can be used to minimize the cost function and recover the registration parameters θ_i. Examples of such approach for rigid registration are given in [Fig. (1)]. Left Ventricle hand-drawn contours (40) from 2/4-chambers view have been considered and registered to the same target.

Medical imaging is an area where quite often global motion is not a proper answer when solving the registration [5]. Local deformations are a complementary component to the global registration model. Dense local motion (warping fields) estimation is an ill-posed problem since the number of variables to be recovered is larger than the number of available constraints. Smoothness as well as other form of constraints were employed to cope with this limitation.

In the proposed framework, a global motion model (τ) is recovered using the mutual information criterion. One can use such model to transform the source shape $\mathcal{D}$ to a new shape $\hat{\mathcal{D}} = \tau(\mathcal{D})$ that is the projection of $\mathcal{D}$ to $\mathcal{S}$. Then, local registration is equivalent with recovering a pixel-wise deformation field that creates visual correspondences between the implicit representation $[\Phi_\mathcal{S}]$ of the target shape $\mathcal{S}$ and the implicit representation $[\Phi_{\hat{\mathcal{D}}}]$ of the transformed source shape $\hat{\mathcal{D}}$.

3 Free-Form Deformations & Local Registration

Such deformation field $L(\mathbf{\Theta}; \mathbf{x})$ can be recovered either using standard optical flow constraints or through the use of warping techniques like the free form deformations method [9], which is a popular approach in graphics, animation and rendering [4]. Opposite to optical flow techniques, FFD techniques support smoothness constraints, exhibit robustness to noise and are suitable for modeling large and small non-rigid deformations. Furthermore, under certain conditions, it can support a dense registration paradigm that is continuous and guarantees a one-to-one mapping.

The essence of FFD is to deform an object by manipulating a regular control lattice P overlaid on its volumetric embedding space. We consider an Incremental Cubic B-spline Free Form Deformation (FFD) to model the local transformation L. To this end,

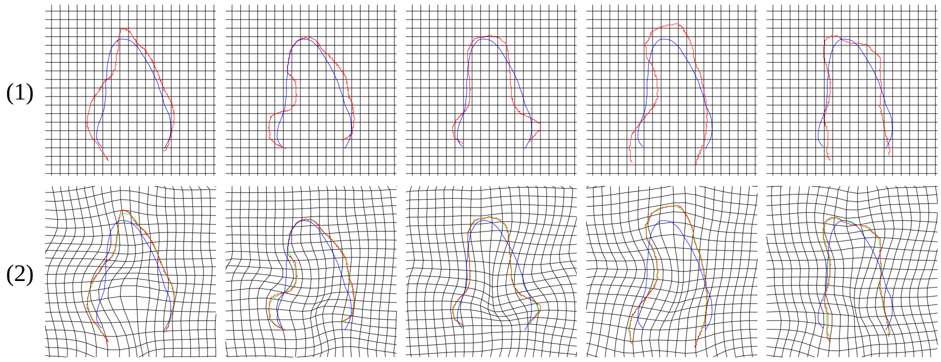

Fig. 2. Local Non-rigid registration using Incremental FFD. (1) initial undeformed grid overlaid on global rigid registration result (blue - mean reference shape), (2) deformed grid to map the reference shape to various training shapes. Each column corresponds to a different trial.

dense registration is achieved by evolving a control lattice P according to a deformation improvement $[\delta P]$. The inference problem is solved with respect to - the parameters of FFD - the control lattice coordinates.

Let us consider a regular lattice of control points

$$P_{m,n} = (P^x_{m,n}, P^y_{m,n});\; m = 1, ..., M, \quad n = 1, ..., N$$

overlaid to a structure

$$\Gamma_c = \{\mathbf{x}\} = \{(x, y) | 1 \leq x \leq X, 1 \leq y \leq Y\}$$

in the embedding space that encloses the source structure. Let us denote the initial configuration of the control lattice as P^0, and the deforming control lattice as $P = P^0 + \delta P$. Under these assumptions, the incremental FFD parameters are the deformations of the control points in both directions (x, y);

$$\mathbf{\Theta} = \{(\delta P^x_{m,n}, \delta P^y_{m,n})\};\; (m, n) \in [1, M] \times [1, N]$$

The motion of a pixel $\mathbf{x} = (x, y)$ given the deformation of the control lattice from P^0 to P, is defined in terms of a tensor product of Cubic B-spline:

$$L(\mathbf{\Theta}; \mathbf{x}) = \mathbf{x} + \delta L(\mathbf{\Theta}; \mathbf{x}) = \sum_{k=0}^{3} \sum_{l=0}^{3} B_k(u) B_l(v) (P^0_{i+k,j+l} + \delta P_{i+k,j+l})$$

where $k = \lfloor \frac{x}{X} \cdot M \rfloor - 1,\; l = \lfloor \frac{y}{Y} \cdot N \rfloor - 1$. The terms of the deformation component refer to (i) $\delta P_{i+l,j+l}$, $(k, l) \in [0, 3] \times [0, 3]$ consists of the deformations of pixel $\mathbf{x}$'s (sixteen) adjacent control points, (ii) $\delta L(\mathbf{x})$ is the incremental deformation at pixel $\mathbf{x}$, and (iii) $B_k(u)$ is the k^{th} basis function of a Cubic B-spline ($B_l(v)$ is similarly defined).

Local registration now is equivalent with finding the best lattice P configuration such that the overlaid structures coincide. Since structures correspond to distance transforms of globally aligned shapes, the Sum of Squared Differences (SSD) can be considered as the data-driven term to recover the deformation field $L(\mathbf{\Theta}; \mathbf{x})$);

$$E_{data}(\mathbf{\Theta}) = \iint_{\Omega} \left(\Phi_{\hat{\mathcal{D}}}(\mathbf{x}) - \Phi_{\mathcal{S}}(L(\mathbf{\Theta}; \mathbf{x}))\right)^2 d\mathbf{x}$$

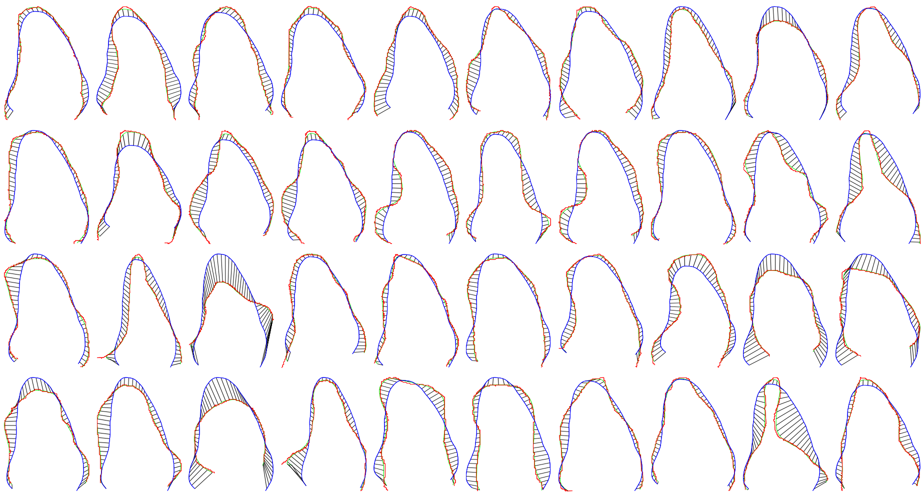

Fig. 3. Examples of establishing correspondences using Incremental FFD. (red) Global registration result, (blue) target mean shape, (dark lines) correspondences for a fixed set of points on the mean shape & (green) the projection of mean shape on each training shape.

The use of such technique to model the local deformation registration component introduces in an implicit form some smoothness constraint that can deal with a limited level of deformation. In order to further preserve the regularity of the recovered registration flow, one can consider an additional smoothness term on the deformation field δL. We consider a computationally efficient smoothness term:

$$E_{smoothness}(\mathbf{\Theta}) = \iint_{\Omega} \left(\left\| \frac{\partial \delta L(\mathbf{\Theta}; \mathbf{x})}{\partial x} \right\|^2 + \left\| \frac{\partial \delta L(\mathbf{\Theta}; \mathbf{x})}{\partial y} \right\|^2 \right) d\mathbf{x}$$

Such smoothness term is based on a classic error norm that has certain known limitations. One can replace this smoothness component with more elaborated norms. Within the proposed framework, an implicit smoothness constraint is also imposed by the Spline FFD. Therefore there is not need for introducing complex and computationally expensive regularization components.

The Data-driven term and the smoothness constraints term can now be integrated to recover the local deformation component of the registration and solving the correspondence problem: $E(\mathbf{\Theta}) = E_{data}(\mathbf{\Theta}) + \alpha E_{smoothness}(\mathbf{\Theta})$, where α is the constant balancing the contribution of the two terms. The calculus of variations and a gradient descent method can be used to optimize such objective function [6]. The performance of the proposed framework on the Systolic Left Ventricle dataset is demonstrated in [Fig. (2)] (FFD grid deformations) and [Fig. (2),(3)] (established local correspondences).

4 Building Compact Representations

Let us now assume the existence of n ground truth examples $\phi_{i=1...n}$ in a training set for a structure of interest[Fig. (1)]. Registering these examples to a common pose and establishing local correspondences is required prior of creating a compact statistical representation. Then Principle Component Analysis (PCA) can be applied to capture

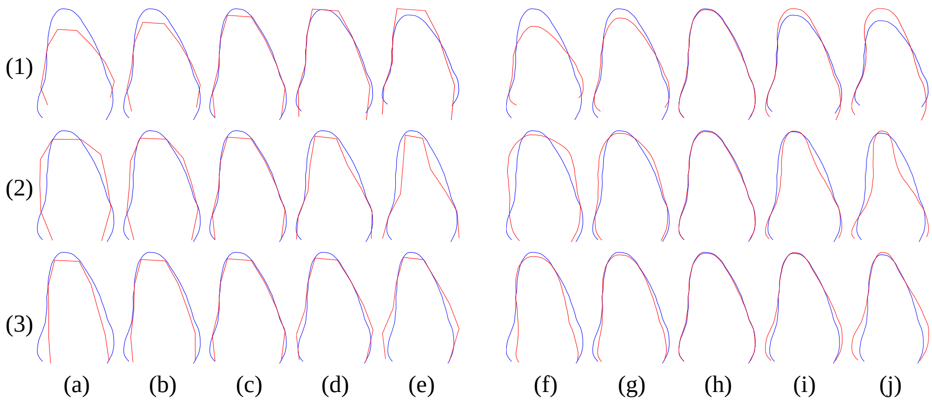

Fig. 4. Principle Component Analysis modeling for the systolic Left Ventricle shapes in Ultrasonic images using the established local correspondences. Changing modes of variation from $-2\sqrt{(\lambda_1)}$ to $2\sqrt{(\lambda_1)}$: (1) first, (2) second, (3) third; (a)-(e) 10-points based model, (f)-(j) 80-points based model.

the statistics of the corresponding elements across the training examples. PCA refers to a linear transformation of variables that retains - for a given number n of operators - the largest amount of variation within the training data, according to: $\phi = \overline{\phi} + \Sigma_{j=1}^{m} b_j\ U_j$, where $\overline{\phi}$ is the mean shape, m is the number of retained modes of variation, U_j are these modes (eigenvectors), and b_j are linear weight factors within the allowable range defined by the eigenvalues.

The most critical part of such analysis process is the representation of the training examples using the same number of elements. Each element corresponds to the same location on the standard atlas of the structure of interest. Simplistic approaches on establishing local correspondences are based on uniform (equal-distance) sampling, parametric approximation of the training set using the same number of basic components, etc. However, these methods require explicit parameterization of the shapes, finding at least one pair of correspondence between landmark points or finding the correspondence between parameterization schemes, which are all difficult problems that do not have straightforwardly extensible solutions in handling structures of high dimensionality and those with complex topology (e.g. multi-parts). Furthermore, without the support of dense optimal local registration, the resulting correspondences from these methods are often non-intuitive and prone to noise.

The proposed global-to-local registration framework can cope with the above limitations. To this end, first all contours are registered to the same target. Such selection introduces bias on the modeling phase. To overcome this limitation, we apply an iterative schema where the mean shape approximated as in [8] can be used as target to perform registration. Once global registration is completed, local correspondences between the reference shape and the examples of the training set are established using the free-form deformation approach. Then according to the desired dimensionality of the model, one can sample the reference shape, use the one-to-one dense local deformation field to recover the corresponding positions within the training set, and to extract the compact representation for the set of training examples using PCA [Fig. (4)]. There is a compromise between the model complexity (number of elements) and the accuracy of the compact statistical representation.

5 Discussion

In this paper we have proposed a novel variational technique for establishing local correspondences between shapes. Registration has been approached in an hierarchical manner. First, a rigid motion model has been determined between the target and the source and then a dense registration field was recovered, supplementary to the global motion model. Shapes were considered in a higher dimension, the space of distance transforms. Such space when combined with mutual information results to a powerful registration paradigm. The use of free-form deformations was considered to recover the local registration component in the space of distance transforms leading to one-to-one mapping between the source and the target. Principal component analysis was considered for modeling of the registered set of training examples. The proposed framework is completely automatic and efficient, it can be used for handling very large training datasets of anatomical structures in arbitrary dimension (2D or 3D). The compact statistical representations (prior models) thus built can be used in a similar manner to the Active Shape Models (ASM) for model search in image segmentation and tracking applications.

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