Application of Particle Filtering to Image Enhancement

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Research Report 05-18
December 2005

Ecole Nationale Des Ponts et Chaussées
Centre d'Enseignement et de Recherche en Technologies de l'Information et Systèmes

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Applications des Filtres à Particules à la Restauration des Images

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Abstract

In this report we propose a novel - assumption-free on the noise model - technique based on random walks for image enhancement. Our method explores multiple neighbors sets (or hypotheses) that can be used for pixel denoising, through a particle filtering approach. This approach associates weights for each hypotheses according to its relevance and its contribution in the denoising process. Toward accounting for the image structure, we introduce perturbations based on local statistical properties of the image. In other words, particle evolution are controlled by the image structure leading to a filtering window adapted to image contents. Promising experimental results and comparison with the state-of-the-art methods demonstrate the potential of such an approach

Résumé

Dans le présent rapport, nous proposons une nouvelle technique de restauration d'images qui s'appuie sur les marches aléatoires et sans hypothèses sur le modèle du bruit. Notre méthode examine différents ensembles de voisins (hypothèses) qui pourront être utilisés pour le débruitage d'un pixel donnée, en utilisant les filtres à particules. Cette approche associe des poids à chaque hypothèse suivant sa pertinence et sa contribution dans le débruitage. Afin de prendre en compte les structures de l'image, nous introduisons des perturbations qui s'appuient sur des propriétés statistiques locales de l'image. En d'autres termes, l'évolution d'une particule est régie par la structure de l'image pour aboutir à une fenêtre de filtrage adaptée au contenu de l'image. Des résultats expérimentaux prometteurs et la comparaison avec l'état de l'art montrent le potentiel de notre approche.

Contents

1 Introduction

In spite of the progress made in the field of image denoising, its still an open issue. In fact, natural images are mixtures of various types of information such as texture, small details, noise, fine structure and homogeneous regions and this makes image filtering a crucial and challenging task. Ideally, a denoising technique must preserve all image element except noise.

Prior art in image denoising consists of methods of various complexity. Local filter operators, image decomposition in orthogonal spaces, partial differential equations as well as complex mathematical models with certain assumptions on the noise model have been considered. The sigma filter method [15], the bilateral [22] filter, morphological operators [25] and the mean shift algorithm [5] are efficient local approaches to image denoising. The first two approaches compute a weighted average of the pixel neighborhood where weights reflects the spatial distance between pixels and also the difference between their intensities. Such methods account to a minimal extend for the image structure and introduce strong bias in process through the selection of the filter operator bandwidth.

Image decomposition in orthogonal spaces like wavelets [17], splines, fourier descriptors and harmonic maps is alternative to local filtering. Images are represented through a class of invertible transformations based on an orthogonal basis. Filtering consists of modifying the coefficients of the transformation space where often the most important ones eliminated. Reconstruction of the image using the new set of coefficients leads to natural denosing. In their origin such methods failed to preserve boundaries a limitation that has been addressed through more careful selection of the orthogonal basis driven from the image structure [6, 13]. Such techniques have good performance when dealing with edges but they fail to preserve small details and texture.

Partial differential equations[1], higher order nonlinear operators [2], and functional optimization [19, 21, 23] have been also considered to address image denoising. The anisotropic diffusion [20] was a first attempt to incorporate image structure in the denosing process. Despite numerous advantages, theoretical justification [3] and numerous provisions of such a method one can claim that it remains myopic and cannot deal with image textures. The mumford-shah framework [19], the total variation minimization [21], the Beltrami flow [12], and other cost functionals of higher order [2] make the assumption that the image consists of a noise-free smooth component and the oscillatory pattern which corresponds to the random noise. Within such a concept constraints at limited scale are also introduced and image is reconstructed through the lowest potential cost function, that is often recovered in an iterative fashion through the calculus of variations. In the most general case such cost functions are not convex and therefore the obtained solution could correspond to a local minimum. Such methods are also myopic and still fail to account for texture patterns despite recent advances [24].

In order to account for image structure [18] an effort to understand the behavior of natural images when seen through a set of orientation and scale selective band-pass operators was made [14, 16]. Central assumption on this effort was that images exhibit differentially Laplacian statistics [16]. Such information is critical to an image denoising approach since it suggests the right way to regularize the problem and design the most efficient algorithm. Despite promising results, such simplistic modeling often fails to capture dependencies in a larger scale as well as account for the presence of repetitive patterns like texture.

To conclude, traditional/state-of-the art techniques are often based on restoring image values based on local smoothness constraints within fixed bandwidth windows where image structure is not considered. Consequently a common concern for such methods is how to choose the most appropriate bandwidth and the most suitable set of neighboring pixels to guide the reconstruction process. In this context, the present work proposes a denoising technique based on multiple hypotheses testing. To this end, the reconstruction process is guided from multiple random walks where we consider many possible neighboring sites in the image and through a particle filtering process, we track the most suitable ones. Furthermore, image structure at a variable local scale is considered through a learning stage that consists of recovering probabilistic densities capturing co-occurrences of visual appearances at scale spaces. Kernels of fixed bandwidth are used to approximate such individual complex models for the entire visual spectrum. Random perturbations according to these densities guide the "trajectories"of a discrete number of walkers, while a weighted integration of the intensity through the random walks leads to the image reconstruction. Such a method is presented in [Fig. (1)].

The reminder of this document is organized in the following fashion; in section 2 we discuss the co-occurrences of image structure learning. Random walks and particle filters are presented in section 3, section 4 will be devoted to the application of the particle filtering to denoising as well as some experimental results and comparisons with the state of the art methods . Finally, we conclude in section 5.

2 Statistics of Natural Images

Understanding visual content has been a constant effort in computer vision with applications to image segmentation, classification, retrieval and coding. Statistical modeling of images aims to recover contextual information at a primitive stage of visual processing chain. Co-occurrence matrices [11] have been a popular method to classification and segmentation of texture images.

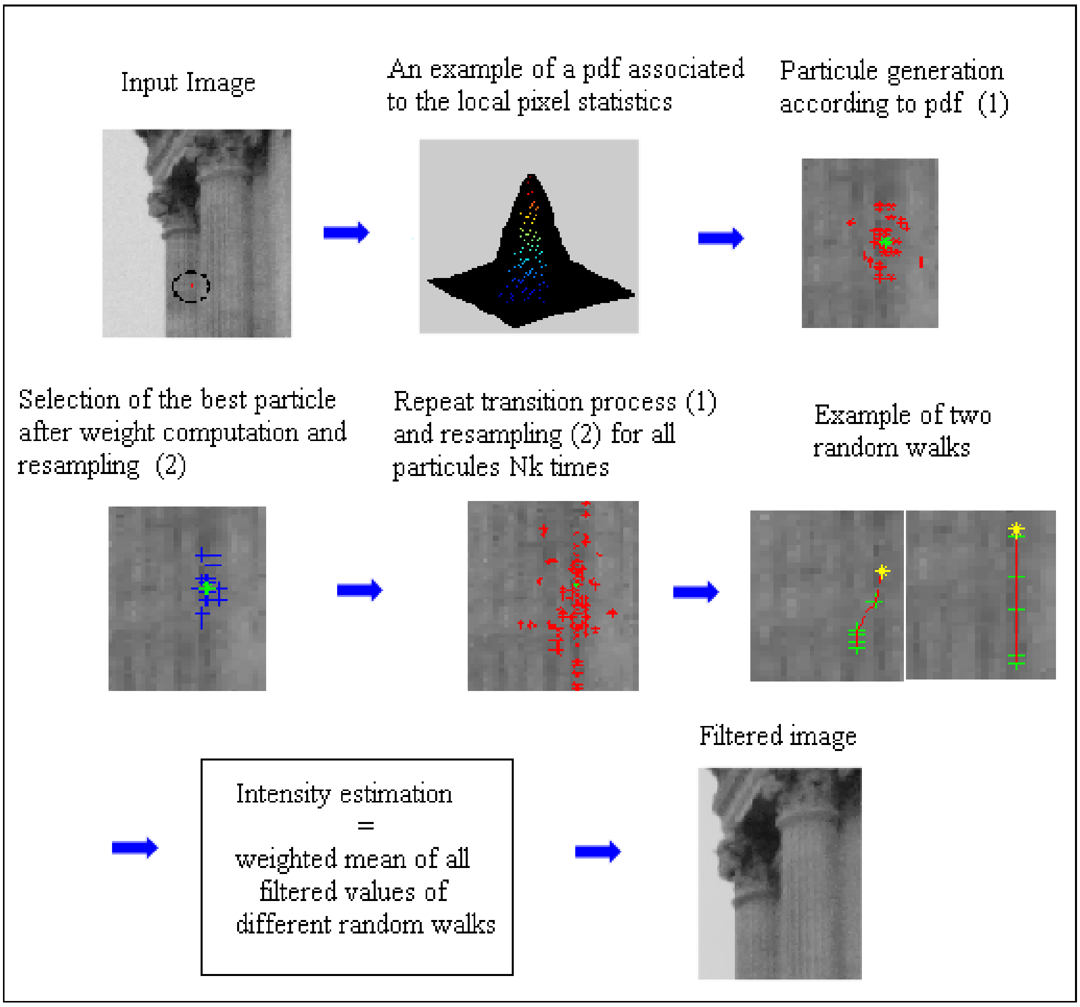

Figure 1: Overview of Random Walks, Constrained Multiple hypotheses Testing and Image Enhancement.

Such a matrix is defined by a distance and an angle, and aim to capture spatial dependencies of intensities. The formal mathematical definition of an element (m, n) for a pair (d, θ) is the joint probability on the image that a m-valued pixel co-occurs with a n-valued pixel, with the two pixels are separated by a distance d and an angle θ:

$$C_{d,\theta}(m,n) = p_{(\mathbf{x},\mathbf{y})\in\Omega}(m,n)\left(I(\mathbf{x}) = m, I(\mathbf{y}) = n, \mathbf{y} - \mathbf{x} = de^{i\theta}\right)$$

with I being the observed image and Ω its domain. In the case of image denoising, the diagonal values of this matrix are of significant importance since implicitly they provide information on the geometric structure of the image. Inspired by such a concept, an intelligent denoising algorithm should be able to extract the most important correlations of local structure from the entire image domain, that is an ill-posed problem. Let us assume the absence of knowledge on the noise model. Then, in order to encode image structure we should seek for an estimate of the posterior

$$p(d,\theta,\mathbf{x}) = pdf(\big\{(d,\theta,\mathbf{x}),\ \text{where } \mathbf{x}\in\Omega, I(\mathbf{x}) = I(\mathbf{x}+de^{i\theta});\big\})$$

that models cross-dependencies at the pixel level. The estimation of such posterior at a local scale in particular when modeling noisy images is an ill-posed problem with enormous complexity. If we assume that images often contain repetitive structures, one can ignore the spatial parameter of such a pdf and seek for an estimate of a global density that captures co-dependencies at global scale, or

$$p_f(d,\theta) = pdf(\big\{(d,\theta) \text{ where } \mathbf{x}\in\Omega, I(\mathbf{x}) = I(\mathbf{x}+de^{i\theta}) = f\big\})$$

To account for pixel values corrupted by noise, the constraint of exact matching could be relaxed , leading to:

$$p_{f,s}(d,\theta) = pdf(\big\{(d,\theta) \text{ where } \mathbf{x}\in\Omega, I(\mathbf{x}) = f, \big[\delta(\mathbf{x},\mathbf{x}+de^{i\phi}) < \epsilon\big] \text{ and } [d<s]\big\})$$

where s is the scale considered for the pdf computation and $\delta(;)$ is a metric that reflects similarity between to pixels in the image. This metric can be a simple distance such as the L^1 or the L^2 norm or more complex measure like correlation, histogram matching, mutual information, etc. In our experiments, we integrated the local variance into the pdf expression. In fact local variance (noted $\sigma(.)$) is a simple primitive capable of describing texture at small scales. The new formulation of pdf is then as follows:

$$\begin{aligned} p_{f,\sigma,s}(d,\theta) =& pdf(\{(d,\theta) \text{ where } \mathbf{x}\in\Omega, I(\mathbf{x}) = f,\\ &\big[\delta(\mathbf{x},\mathbf{x}+de^{i\theta}) < \epsilon_1\big]\,;\big[\eta(\sigma(\mathbf{x}),\sigma(\mathbf{x}+de^{i\theta})) < \epsilon_2\big] \text{ and } [d<s]\big\})\end{aligned}$$

As far as scale is concerned, different methods can be used to self-determine the scale like in the case of co-occurrence matrices. In the most general case we can assume scales of variable length that are self-adapted to the image structure. One can pre-estimate such *pdf* from the image using its empirical form.

However, $p_{f,s}(d,\theta)$ aims to capture information of different structure, it describes spatial relation between similar patches in the image. Therefore, $p_{f,s}(d,\theta)$ is highly non-linear and its approximation using standard assumptions like mixture of gaussians is highly unrealistic. Non-parametric kernel-based density approximation strategies [26] like parzen windows is an emerging technique to model highly non-linear structures.

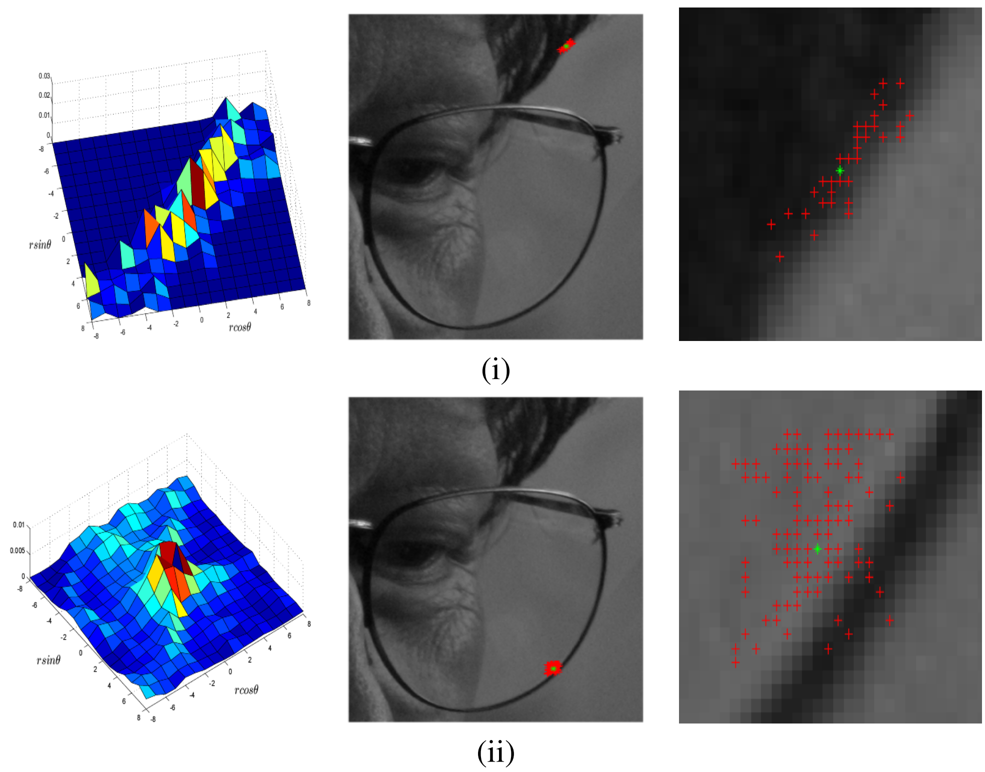

Figure 2: Two pdf distribution $p_{f,\sigma}(d,\theta)$ for different values of f ans σ (top($f = 39, \sigma = 11.67$), bottom($f = 96, \sigma = 3.55$), and sample generation according to these pdf (red pixel)for two different positions

Let $\{\mathbf{x}_i\}_{i=1}^{M}$ denote a random sample with probability density function p. The fixed bandwidth kernel density estimator consists of:

$$\hat{p}(\mathbf{x}) = \frac{1}{M}\sum_{i=1}^{M} \mathrm{K}_{\mathbf{H}}\left(\mathbf{x}-\mathbf{x}_i\right) = \frac{1}{M}\sum_{i=1}^{M} \frac{1}{\|\mathbf{H}\|^{1/2}} \mathrm{K}\left(\mathbf{H}^{-1/2}(\mathbf{x}-\mathbf{x}_i)\right)$$

where $\mathbf{H}$ is a symmetric definite positive - often called a bandwidth matrix - that controls the width of the kernel around each sample point $\mathbf{x}_i$. Gaussian kernels are the most common selection of such an approach and that is what was considered in our case to approximate $p_{f,s}(d,\theta)$. Once such *pdf* has been constructed from the image, we are able for a given image position $\mathbf{x}$ and an observation ($f = I(\mathbf{x})$,σ) to generate a number of hypotheses for the most prominent position of the related image structure ([Fig. 2)].

One can consider now the problem of image denoising for a given pixel as a tracking problem in the image domain. Thus, given a starting position (pixel itself), the objective is to consider a feature vector that upon its successful propagation along similar image structure, is able to remove and recover the original image value. To this end, we define

- a feature vector, that defines the current state of the reconstruction process s_t,
- an iterative process to update the density function, to predict the next state,
- a measure of quality of a given hypothesis (feature vector) with respect to the image data.

with $\left[s_t = (\mathbf{x}_t, \hat{I}(\mathbf{x})\right]$ being the state vector at a given time t. This state vector corresponds to the candidate site than can be used in filtering process and the reconstructed value induced by this site. The statistical interpretation of such an objective refers to the introduction of a probability density function (pdf) that uses previous states to predict possible new positions and image features to evaluate the new position. The multiple hypotheses generation could be done in a number of fashions. Sequential Monte Carlo is are well known techniques that associate evolving densities to the different hypotheses, and maintains a number of them. Particle filters are popular techniques used to implement such a strategy.

3 Bayesian Tracking, Particle Filters & Multiple hypotheses Testing

The Bayesian tracking problem can be simply formulated as the computation of the present state s_t pdf of a system, based on observations from time 1 to time t $z_{1:t}$: $p(s_t|z_{1:t})$. Assuming that one can have access to the priori pdf $p(s_{t-1}|z_{1:t-1})$, the posteriori pdf $p(s_t|z_{1:t})$ can be computed from Bayes' rule:

$$p(s_t|z_{1:t}) \;=\; \frac{p(z_t|s_t)p(s_t|z_{1:t-1})}{p(z_t|z_{1:t-1})},$$

where the prior pdf is computed via the Chapman-Kolmogorov equation

$$p(s_t|z_{1:t-1}) \;=\; \int p(s_t|s_{1:t-1})p(s_{t-1}|z_{1:t-1})ds_{t-1},$$

and

$$p(z_t|z_{1:t-1}) \;=\; \int p(z_t|s_t)p(s_t|z_{1:t-1})ds_{t-1}$$

The recursive computation of the priori and the posteriori pdf leads to the exact computation of the posterior density. Nevertheless, in practical cases, it is impossible to compute exactly the posterior pdf $p(s_t|z_{1:t})$, which must be approximated.

Particle filters, which are sequential Monte-Carlo techniques, estimate the Bayesian posterior probability density function (pdf) with a set of samples. Sequential Monte-Carlo methods have been first introduced in [9, 27]. For a more complete review of particle filters, one can refer to [10, 7].

Particle filtering methods approximate the posterior pdf by M random state sample $\{s_t^m, m = 1..M\}$ associated to M weights $\{w_t^m, m = 1..M\}$, such that

$$p(s_t|z_{1:t}) \approx \sum_{m=1}^{M} w_t^m \delta(s_t - s_t^m).$$

Thus, each weight w_t^m reflects the importance of the sample s_t^m in the pdf.

The samples s_t^m are drawn using the principle of Importance Density [8], of pdf $q(s_t|s_{1:t}^m, z_t)$, and it is shown that their weights w_t^m are updated according to

$$w_t^m \propto w_{t-1}^m \frac{p(z_t|s_t^m)p(s_t^m|s_{t-1}^m)}{q(s_t^m|s_{t-1}^m, z_t)}. \tag{1}$$

This equation shows that particle weights are updated using two mainly informations : the observation pdf which reflects the likelihood of seeing an observation z_t knowing the state s_t and the transition model which control the evolution of a particle state. The *sampling importance resampling* algorithm (SIR) consists in choosing the prior density $p(s_t|s_{t-1})$ as importance density $q(s_t|s_{1:t}^m, z_t)$. Doing so, equation (1) becomes simply

$$w_t^m \propto w_{t-1}^m p(z_t|s_t^m), \tag{2}$$

To sum up particle filtering consists of three main steps:

- particle drawing according the transition law $p(s_t^m|s_{t-1}^m)$
- computation of the likelihood of observations generated by the particle $p(z_t^m|s_t^m)$
- weight updating according to $w_t^m \propto w_{t-1}^m p(z_t|s_t^m)$

After several steps a degeneracy issue occurs, such that all weights but few become null. In order to keep as many samples as possible with respectful weights, a resampling is necessary. Different resampling processes exist. The SIR algorithm consists in selecting the most probable samples in a random way, potentially selecting several times the same sample.

In a first step, the weights' cumulative density function (cdf) $\{c_j\}$ would be computed. Then, the selection step would consist in choosing a random number $0 < r < 1$, and finding the smallest j such that $c_j < r$. The selected state would then be s_t^j. This selection would be repeated N times, to select N samples. Finally,

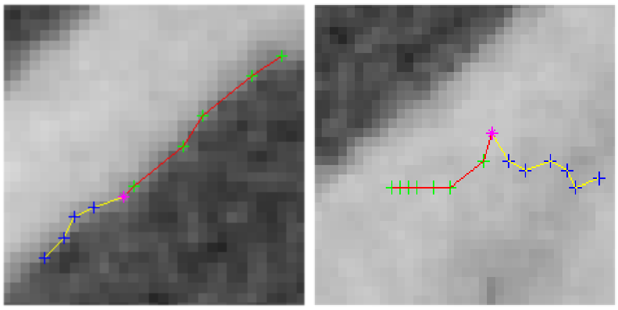

Figure 3: Two examples of random walks where the origin pixel is on the border (left) or in an homogeneous region (right)

a random vector would be added to each one of these samples. The SIR algorithm is the most widely used resampling method because of its simple formulation, its easy implementation and the fact that fewer samples are required, and thus the computational cost may be reduced with respect to other resampling algorithms. An example of propagation of multiple hypotheses is shown in the figure 3 for two different origin pixels.

4 Random Walks and Image Denoising

We now consider the application of such non linear model to image denoising. Thus, given an origin pixel $(\mathbf{x})$ reconstruction is equivalent to recovering a number of "‘random"’ positions $(\mathbf{x} = \mathbf{x}_0, \mathbf{x}_1, ..., \mathbf{x}_\tau)$ with similar properties to $\mathbf{x}$ to reconstruct the corrupted origin value $(I(\mathbf{x}))$. The set of final trajectories of each particle and their corresponding weights will represent the "‘filtering window"’. To this end, the use of "‘constrained"’ multiple hypotheses will be considered. This approach requires the definition of a perturbation model as well as a likelihood measure that reflects the contribution of a trajectory to the denoising process.

4.1 Likelihood measure

Measuring similarities between image patches has been a well studied problem in computer vision. In the case of denoising an ideal filtering approach should consider pixels with the exact same value. Within the proposed approach filtering is done in a progressive fashion and therefore a need exists to measure the contribution of a new element in the filtering process. Parallel to that, each particle corresponds to a random walk where a certain number of pixels have been selected and contribute to the denoising process. Therefore, we define two metrics, one that accounts for the quality of potential additions and one for the intra-variability of the trajectories.

- The L^1 error-norm between local neighborhoods centered at the current position $\mathbf{x}_t$ and at the origin pixel $\mathbf{x}$.

  $$D_{sim}(t) = \frac{1}{(2W+1)^2} \sum_{\mathbf{v} \in [-W,W] \times [-W,W]} |I(\mathbf{x}+\mathbf{v}) - I((\mathbf{x_t}+\mathbf{v})|$$

  where W is the bandwidth which must be carefully selected to get a reliable measure of similarity while being computationally efficient.

- In order to account for the intra-variability of the trajectories, we consider the variance, centered at the origin value,

  $$D_{intra}(t) = \frac{1}{t} \sum_{\tau=0}^{t} (I(\mathbf{x}_\tau) - I(\mathbf{x}))^2$$

  that measure the "uniformity" of the trajectory and could also be determined within a larger neighborhood (not at the pixel level). This terms insures edges and fine structure enhancement since random walks with small intra-variability are favored.

These two metrics are considered within an exponential function to determine the importance/contribution of a new sample under consideration given the prior state of the walk.

$$w_t = e^{-\frac{D_{sim}(t)+D_{intra}(t)}{2\sigma_w^2}} \tag{3}$$

The next step consists of defining an appropriate strategy for samples perturbation.

4.2 Perturbation Model & Online Adaptation

Within the first stage of our approach we have introduced the notion of learning image structure statistics $p_{f,\sigma}(;)$. Such a distribution can be used to guide the perturbation model at a given time t, $\left[p_{I(\mathbf{x}_t),\sigma(I(\mathbf{x}_t))}\,(;)\right]$.

During the construction of $p_{f,\sigma}(d,\theta)$ information from different structural origins was used. Pixels, image regions or image structure with similar values (f,σ) often correspond to different spatial-driven local structure. On the other hand, upon completion of the filtering process and given the constraint of local perturbations, more and more structural information are added to the "'random"' walk. It is adequate to use such information to update the non-parametric form of the

$p_{f,\sigma}(d,\theta)$. Simple forgetting mechanisms can be used to update such density according to :

$$p^t_{f,\sigma}(\pi) = \underbrace{\alpha\,\frac{1}{M_p}\sum_{i=1}^{M_p} \mathrm{K}_{\mathbf{H}}\left(\pi - \pi_i\right)}_{prior\ density} + \underbrace{(1-\alpha)\,\frac{1}{t}\sum_{\tau=0}^{t} \mathrm{K}_{\mathbf{H}(\mathbf{w_t})}\left(\pi - \hat{\pi}_\tau\right)}_{online\ local\ density} \qquad (4)$$

where M_p is the number of perturbation samples, $\hat{\pi} = (\hat{d},\hat{\theta})$ is the perturbation that must be applied to a particle during the transition step, and w_t is the weight associated to it. This leads to a variable bandwidth kernels where perturbations of limited interest do not contribute to the evolving density. In other words in this transition pdf, the first term of the expression corresponds to prior density introduced in section 2 while the second is an update term which learns the most interesting perturbations and encodes them in the pdf formulation.

4.3 Implementation and Validation

In this section we will be concerned about the application of the particle filtering process to denoising. To this end, for each pixel $\mathbf{x}$ of the image, we generate N number of particles by applying N perturbations to the initial position $\mathbf{x}$. Then, each particle is propagated using a perturbation driven from the conditional distribution of the image statistics described by equation (4). The process is repeated for (T) iterations. In each step of the process, we associate to each random walk a weight according to the likelihood measure defined in expression (3). We define then the walk value $\hat{I}^m_t(\mathbf{x})$ as the average value along the walk. It corresponds to the estimated value of the noisy pixel proposed by the "random walk" m :

$$\hat{I}^m_t(\mathbf{x}) = \frac{1}{t}\sum_{\tau=0}^{t} I(\mathbf{x}^m_\tau)$$

Linear combination of the hypotheses weights and the corresponding image denoised walk values is used to produce the current state of the process:

$$\hat{I}_t(\mathbf{x}) = \sum_{m=0}^{N} w^m_t \hat{I}^m_t(\mathbf{x})$$

In order to avoid degeneration of samples, as well as use with maximum efficiency all hypotheses, a frequent resampling process is used to address such a demand. In practice we use (N=30) particles, with (T=10) pixels contributing to each walk. To illustrate the random walks filtering an overview of the hole process is presented in figure 1

	Original Image	Bilateral	NLmean	Random Walk
Canon 1Ds (ISO 1000)	13.55	13.16	13.32	12.9
Nikon D70 (ISO 1250)	22.60	22.44	22.6	22.42
Canon 1Ds (ISO 1250)	14.16	13.86	14.16	13.69
Nikon D70 (ISO 1600)	23.35	23.24	23.35	23.21
Canon 300D(ISO 1600)	17.94	17.7	17.94	17.54

Table 1: Method performance in noise reduction of calibration pattern relative to different digitals cameras; values correspond to the reduction of the standard deviation relative to a uniform patch.

Towards the validation of the method, we used images of unknown noise models and have compared our method with well known filtering techniques such as the bilateral filter [22] and the Non Local Mean [4] approach. Some qualitative results and comparisons are shown in [Fig. (4 and 5)]. As for quantitative comparaison, number of methods have been used in the literature for such comparisons, given the absence of knowledge on the noise model we have considered calibration patterns and studied the behavior of the tested approaches on these patterns when observed from different digital cameras. We focus on the noise reduction on these patterns which is equivalent to the reduction of the standard deviation relative to a uniform patch. Table 4.3 shows the performance of each filtering technique in terms of noise reduction for different digital camera models. Considering this criteria, results show that our method outperforms the other techniques. This is explained by the fact that in absence of texture or structure the Random walk acts as an isotropic filtering. The bilateral filter consider the pixel location while denoising which limit the influence of distant pixel in the filtering process. The non local mean,uses to compute neighborhood similarity the $L2$ distance which is more sensitive to outliers then the $L1$ distance we use.

As for qualitative results,we selected for each method the most suitable parameters that gives the best compromise between noise reduction and detail preserving. If we consider the method noise images which corresponds to the difference between the original and the filtered image (see fig 4), we notice that the random walk filtering produces better results than the bilateral filtering in terms of texture and small details preserving. This is explained by the fact that in our approach we make a structure tracking and we integrate information about image statistics in the model. The NL mean technique achieves the best results in term of small detail preserving since its method noise contains less image information then the two other techniques. This is due to the fact that the NLmean algorithm scans all image pixels to select the best candidate while denoising a given pixel and this make it very slow in terms of computation time.

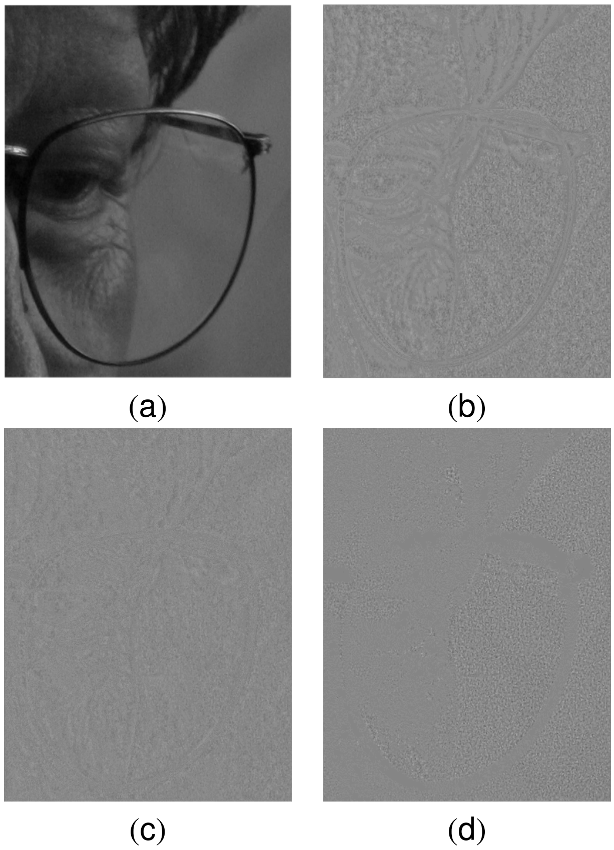

Figure 4: Original image (a) and method noise for Bilateral filtering (b) , Random walk (c) , NLmean (d)

5 Conclusions

In this paper we have proposed a novel technique for image filtering. The core contribution of our method is the selection of an appropriate walk in the image domain towards optimal denoising. Such concept was implemented in a non-exclusive fashion where multiple hypotheses are maintained. The use of monte-carlo sampling and particle filters was considered to inherit such a property in the method. Furthermore, inspired by co-occurrence matrices we have modeled global image structure towards optimization on the selection of trajectories of the multiple hypotheses concept. To further adapt the method to the image structure such modeling was updated on line using local structure. Promising experimental results demonstrate the potentials of our approach.
Computational complexity is a major limitation of the method. The use of smaller number of hypotheses could substantially decrease the execution time. Improving the learning stage and guiding the particles to the most appropriate directions is

a short term research objective. To this end, we would like to provide techniques capable of selecting the scales of each operator. Furthermore, we would like to consider kernels of variable bandwidth when recovering the non-parametric form of the learned distribution that are more efficient to capture image structure. More long term research objectives refer a better propagation of information within trajectories. Particle filters is a fairly simple approach that mostly propagates the mean value and the weights. The propagation of distributions can better capture the importance of the trajectories as well as the effect of new additions. In addition to that, geometric constraints on the "'walks"' could also improve the performance of the method in particular when texture is not present.

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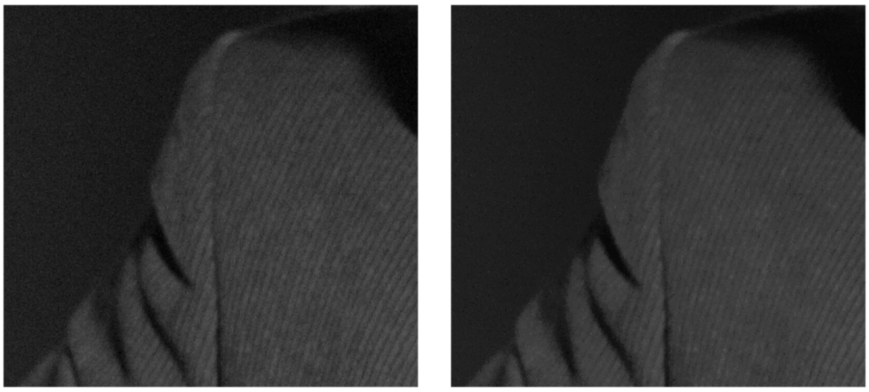

(i) (ii)

(i) (ii)

Figure 5: Results of random walks filtering on natural images where texture is corrupted by noise; (i) Input image, (ii) Denoised Image.