Computation of the electrical potential inside the nerve induced by an electrical stimulus

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***Abstract*— The aim is to investigate the activation conditions of the different nerves which control the bladder. The selective stimulation of the nerve fibers depends on electrode configuration and intensity of applied current. The goal of this study is to compute the electrical potential inside the nerve due to an applied boundary currents. A symmetrically cylindrical model, representing the geometry and electrical conductivity of a nerve surrounded by a connective tissue and a cuff is used. In the quasistatic approximation, the problem can be modeled by a Poisson equation with Neumann boundary conditions. A symmetric boundary integral formulation is discretized using mixed finite elements. We can thus compute an electrical potential distribution depending on the electrode configuration and the applied current inside a nerve. Our results show that the distribution of the electrical potential inside a nerve or a fascicle depends on the geometry of the electrode and the shape of the applied current.**

I. INTRODUCTION

Functional Electrical Stimulation (FES) provides a way to restore movement of paralyzed limbs by activating the efferent somatic axons. Indeed they innerve the striated muscle fibers that can thus generate force when stimulated. FES can also be used to activate other target organs such as the Detrusor smooth muscle for bladder control. The principle remains the same *i.e.* firing motor axons of the desired target muscles. However, within a nerve composed of different types of axons innervating muscle fibers - smooth, slow and fast striated, one challenge to get efficient implanted FES system is to provide a way to selectively activate these fibers; for instance to stimulate the Detrusor (smooth muscle) without stimulating the striated sphincter (innervated by the same sacral root) for bladder emptying, stimulating slow fibers instead of fast fibers in a striated muscle to limit fatigue effect for movement restoration. In the literature, some experimental tests of FES [1], [2], [3] show that the fiber selectivity is possible depending on the geometry of the electrodes and the shape of the stimulus but these results stay empirical and do not give an explicit form of the potential inside a nerve. Some studies concerning the modeling of the interaction between nerve fibers and the stimulating electrodes can be found. Among these, we can cite [4] which examines the behavior of axons excited by electrical fields. In [5], simulations show that the diameter dependency of nerve fiber recruitment is influenced by the electrode geometry. The results, given in [6], predict that the use of intraneural or even intrafascicular electrodes is necessary for selective stimulation of fascicles not lying on the surface of the nerve. Authors in [7] conclude that a transverse tripole activates superficial nerve fibers in a more selective way than other configurations (monopole, bipole, ...). Using a volume conductor model to compute the electrical potential distribution inside a tripolar cuff electrode and a human fiber model to simulate the fiber response to stimulation, [8] determines a minimal quantity of charge per pulse which is needed for selective nerve stimulation depending on pulse shapes. Our aim is to propose a numerical model of nerve-cuff electrode which will be used to study interactions between nerve fibers and electrode during a FES. This model will be used to determine the optimal geometric parameters for a multipolar cuff electrode for selective neural stimulation. This kind of study has been investigated in [9], but only considering a 2-D conductor model. In our investigations, we consider a 3-D problem and we use a numerical method inspired from the forward problem of Electrical Impedance Tomography (EIT), *i.e.* the computation of an electrical potential due to an applied boundary current. The forward problem of EIT has been solved in [10], [11] using a symmetrical version of the boundary element method (BEM). In the study presented in this article, we were interested in the validation of our numerical model in a simple case, thus the nerve was considered as an isotropic medium. The goal is to compute an electrical potential inside a nerve. Knowing the potential and the current densities on the one hand and the fiber excitation thresholds on the other hand, we can determine which type of fiber has been stimulated according to the selected currents and the geometry. Using the Poisson equation, we introduce our physical model of nerve-cuff electrode. Then, the numerical model based on the BEM, used to compute the electrical potential inside a nerve, is described. We finish giving some results corresponding to different cases of stimulation.

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II. METHODS

A. Physical model

The electrical potential generated by a cuff electrode in a volume conductor satisfies the Poisson equation, obtained from the Maxwell equations in the quasi-static case :

$$\left\{ \begin{array}{rl} \nabla.(\sigma\nabla V) = & 0 \\ \sigma\dfrac{\partial V}{\partial n} = & j, \end{array} \right. \tag{1}$$

where j (A) is the applied current and V (V) is the unknown electrical potential. The conductivity σ (S/m) is piecewise constant *ie* each medium has its own conductivity. In the anisotropic case, the conductivity is represented by a diagonal tensor in cylindrical coordinates:

$$\sigma = \begin{pmatrix} \sigma_r & & 0 \\ & \sigma_r & \\ 0 & & \sigma_z \end{pmatrix} \tag{2}$$

We restrict this study to the isotropic case ($\sigma_r = \sigma_z = \sigma$). The conductivity values are provided in Table I.

A nerve consists in a number of nerve fiber bundles called fascicles. Our geometrical model of a stimulated nerve is cylindrically symmetric nerve with one fascicle of length $L = 80$ mm. Its shape is sketched in Fig.1 and its transversal section in Fig.2. Between the inner and outer cylinders lies the connective tissue (thickness $e = 0.25$ mm). We denote $r = 1$ mm the radius of the fascicle. A stimulation current is applied with a multipolar cuff electrode compound of two anodes and one cathode. The applied current j on the cuff electrode must respect the rule:

$$\int_{electrodes} j = 0. \tag{3}$$

TABLE I

CONDUCTIVITY OF EACH TISSUE (INSPIRED FROM [9]).

Tissue	Conductivity σ (S/m)
fascicle	0.6
connective tissue	1.7

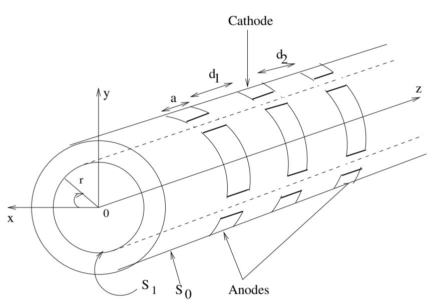


Fig. 1. Schematic design of the nerve-cuff electrode geometry

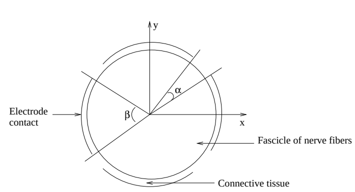


Fig. 2. Transversal section of the nerve-cuff electrode geometry

In Fig. 2 one sees that an anode or a cathode has four contacts. The length of an anode or a cathode along the z axis is denoted a. The distance between one anode and the cathode along the axis z is $d_1 = d_2 = d = 5$ mm in the cases presented in this paper. The angular width of each contact is $\beta = \dfrac{4\pi}{10}$ and the angular distance between two contacts is $\alpha = \dfrac{\pi}{10}$.

B. Numerical model

Numerically, (1) is solved using a symmetrical BEM. The BEM based on a boundary integral equation permits to compute the potential distribution on the surface of a piecewise homogeneous isolated conductor of arbitrary shape, it has been adapted by [11] for the modelisation and simulation of the forward problem of EIT. A detailed description of this method is given in [10]. The conductivity is supposed to be piecewise constant *ie* each surface has its own conductivity. Thus, the conductivity can be factorized in (1) and a Laplace equation is obtained in each volume:

$$\Delta V = 0, \tag{4}$$

with the following boundary conditions at the interface S_1

$$V_{S_1}^{-} = V_{S_1}^{+}, \tag{5}$$

$$\sigma_1 \left(\frac{\partial V}{\partial n}\right)_{S_1}^{-} = \sigma_0 \left(\frac{\partial V}{\partial n}\right)_{S_1}^{+}, \tag{6}$$

where $(-)$ and $(+)$ symbolize the inside and the outside of the surface S_1. In the case presented in this paper, σ_1 corresponds to the conductivity of the fascicle and σ_0 to the connective tissue. The unknown values V_{S_1} and $\left(\sigma\dfrac{\partial V}{\partial n}\right)_{S_1}$ are represented by v_i and $(\sigma\partial_n v)_i$ in a discretized form. In the BEM used here, the formula of the electrical potential V at any point p inside the nerve is computed using

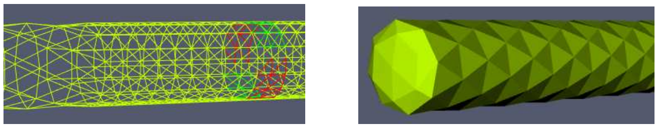
$$\begin{aligned} V(p) = \quad & \sum_i \left(\frac{\|p - p_{T_i}\|}{4\pi} \int_{T_i} \frac{\Phi_i(p)}{\|p - p'\|^3} ds(p') \right) v_i \\ & + \frac{1}{4\pi} \sum_i \left(\int_{T_i} \frac{ds(p')}{\|p - p'\|} \right) (\sigma\partial_n v)_i \,, \end{aligned} \tag{7}$$

where T_i are the triangles of a mesh representing S_1, p' is a point of T_i and p_{T_i} the barycenter of T_i. s is a surface of T_i. Φ_i is the piecewise linear finite element associated to triangle. Surfaces S_0 and S_1 are meshed using finer triangles close to the electrodes. We give an image of the surface S_0 showing the meshes (Fig. 3(a)). The image (Fig. 3(b)) shows the closing of the mesh at the end of the cylinder. As an illustration, Fig. 4 shows the electrical potential on the surface of the nerve.

III. RESULTS

A. Validation

The results given by the numerical scheme can be validated by comparison with analytical solution of the Poisson



(a) (b)

Fig. 3. Zoom of the surface S_0 of the nerve-cuff model. (a) shows that the mesh is finer close to the electrodes. (b) shows that there is a mesh continuity in the boundary of the model.

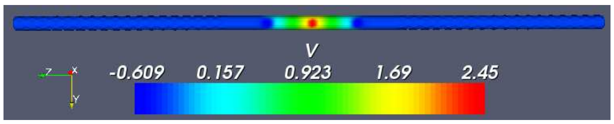


Fig. 4. The upper inset shows the electrical potential distribution at the surface of the nerve model. The lower inset gives the values of the electrical potential.

equation (1) obtained for cylindrically symmetric current injection. Let us assume that the potential V is of the form $V(r,\theta,z) = R(r)Q(\theta)Z(z)$, writing the Laplacian in cylindrical coordinates, (4) becomes

$$\Delta V = \frac{r}{R(r)}\frac{d}{dr}\left(r\frac{dR(r)}{dr}\right) + \frac{1}{Q(\theta)}\frac{d^2Q(\theta)}{d\theta^2} + \frac{r^2}{Z(z)}\frac{d^2Z(z)}{dz^2} = 0 \tag{8}$$

Using boundary conditions, the general solution of (8) is

$$V(r,\theta,z) = \sum_{m,n}\left[A_{m,n}I_n(m\omega r) + B_{m,n}K_n(m\omega r)\right]e^{\pm in\omega z}e^{\pm im\theta}, \tag{9}$$

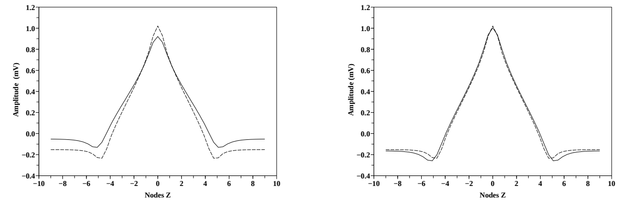
with the constants A and B for each interface are determined from the boundary conditions, I_n and K_n are the modified Bessel functions of the first and the second kind respectively and of order n.

In the following, some results are presented for different values of applied current on the cuff electrode. We present here results corresponding to two types of applied current: In the first case, one contact among four has been used to apply a current and in the second case, a current is applied on all contacts (see Table II for the values). The

TABLE II
DIFFERENT APPLIED CURRENTS

Cases	Cathodic current (μA)	Anodic current (μA)
1	-1,0,0,0	0.5,0,0,0
2	-1,0.5,-1,0.5	0.5,-0.25,0.5,-0.25

shape of the electrical potential on the surface of the nerve computed numerically is close to the solution (9) of the Poisson equation in the cylindrical coordinates (see Fig. 5(a)). A small difference exists in the results, which can be explained from the truncation or the approximation of the numerical values. With a linear regression, the numerical results match the analytical results (see Fig. 5(b)).



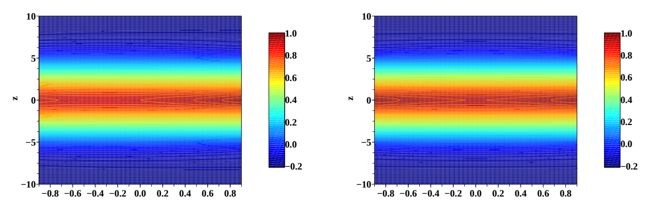
(a) (b)

Fig. 5. Comparison of the electrical potential V on the nerve surface (S_1). The current stimulus corresponds to case 1 and electrode contact size is $a = 1$ mm. Continuous line corresponds to numerical results and dashed line corresponds to theoretical results. (a) without linear regression and (b) with linear regression.

B. Numerical simulations

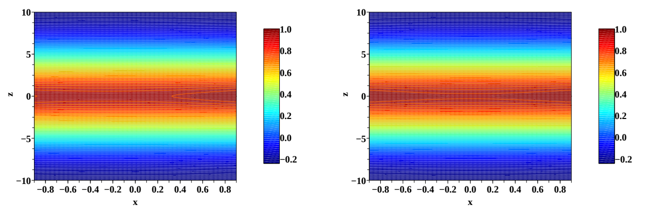
In the following, the colorbar represents the normalized values of the electrical potential (1 corresponds to the maximal value). Figures (6) and (7) show the distribution of the electrical potential values according to a longitudinal section of the nerve with $y = 0$. These figures correspond to a section of the nerve close to the position of the electrodes. We see that the electrical potential propagates symmetrically in two opposite directions. For each symmetrical value of z, there is an homogeneous distribution of the potential according to the x axis of the nerve except around $z \in [-1.2, 1.2]$. This difference leads to disappear if the length a of the electrode contact increases and the applied current is symmetrical (see Fig. 7(b)).

Figures (8) and (9) show the distribution of the electrical potential values according to the transversal section of the nerve. These figures correspond to the section of the nerve at the position of the cathode ($z = 0$). These results provide that the electrical potential increases with the number of stimulation contacts and the length a of the electrode contact. Our results go in the direction of [2], where the authors perform experiments on the cat sciatic nerve and conclude that the application of multiple contact was succesfully used to effect selective activation of fascicles inaccessible with a single contact stimulation.



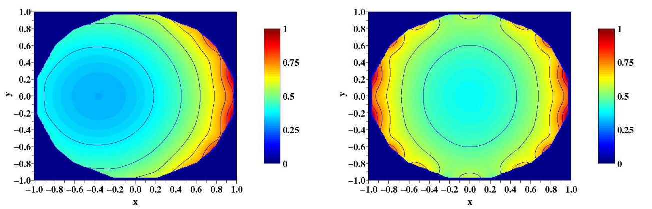
(a) (b)

Fig. 6. Distribution of the electrical potential V according to the longitudinal section of the nerve ($y = 0$), size of electrode contact $a = 1$ mm. (a) and (b) corresponds respectively to case 1 and case 2 of applied current. The maximal value of the electrical potential V_{max} is in the case (a) $V_{max_a} = 0.688$ mV and in the case (b) $V_{max_b} = 0.671$ mV.



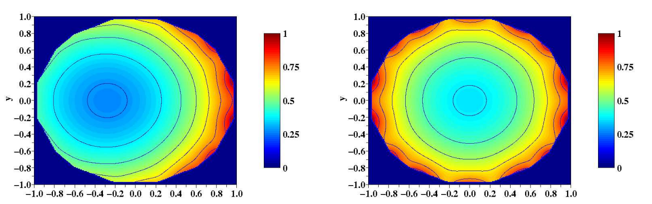
(a) (b)

Fig. 7. Distribution of the electrical potential V according to the longitudinal section of the nerve ($y = 0$), size of electrode contact $a = 3$ mm. (a) and (b) corresponds respectively to case 1 and case 2 of applied current. The maximal value of the electrical potential V_{max} is in the case (a), $V_{max_a} = 2.629$ mV and in the case (b), $V_{max_b} = 2.6$ mV.



(a) (b)

Fig. 8. Distribution of the electrical potential V according to the transversal section of the nerve at $z = 0$, size of electrode contact $a = 1$ mm. (a) and (b) corresponds respectively to case 1 and case 2 of applied current. The maximal value of the electrical potential V_{max} is in the case (a), $V_{max_a} = 0.72$ mV and in the case (b), $V_{max_b} = 0.70$ mV. At $x = y = 0$, $V = 0.639$ mV.



(a) (b)

Fig. 9. Distribution of the electrical potential V according to the transversal section of the nerve at $z = 0$, size of electrode contact $a = 3$ mm. (a) and (b) corresponds respectively to case 1 and case 2 of applied current. The maximal value of the electrical potential V_{max} is in the case (a), $V_{max_a} = 2.66$ mV and in the case (b), $V_{max_b} = 2.63$ mV. At $x = y = 0$, $V = 2.548$ mV.

IV. CONCLUSIONS AND FUTURE WORKS

In this paper, a nerve-cuff electrode model has been presented. We have used an approach inspired from the EIT forward problem and the BEM numerical method. Some preliminary results of the electrical potential distribution inside a fascicle have been given. It will be interesting to test the anodal blocking concept by taking the distance $d_1 \neq d_2$ between the cathode and each anode. Indeed, asymetric cuff with multiphasic current can block the spike propagation in the afferent or efferent way. Besides, the same principle may be used to block the propagation of fast fiber action potentials and then promote slow fiber activation. This numerical model should be used to optimize the stimulation electrode configuration and the shape of the applied current. These results should be used to improve the technology of the FES. This technology leads to conceive and to test multiple implanted functional electrical stimulation devices [12]. We are currently working on the inclusion of the anisotropy of the different media of the nerve. We plan to validate these results through in vitro (isotropic saline solution), and in vivo (on the sciatic nerve of rabbit) experiments. In the last case, intrafascicular electrodes [13], [14] may be used to measure potential and verify the firing of a restricted pool of axons. The functional effect, *i.e.* selective activation of afferent - efferent pathways, slow and fast fibers may be studied through the distal measurements on the nerve of the action potentials. After the validation of the model, the challenge will be the optimisation on one hand, of the design of multipolar electrode and on the other hand of the current waveforms.

REFERENCES

[1] W.M. Grill and J.T. Mortimer, ”Stimulus Waveforms for Selective Neural Stimulation”, *IEEE Engineering in Medecine and Biology*, vol. 95, 1995, pp 375-385.

[2] M.D. Tarler and J.T. Mortimer, ”Selective and Independant Activation of Four Motor Fascicles Using a Four Contact Nerve-Cuff Electrode”, *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 12, No 2, 2004, pp 251-257.

[3] K. Yoshida and K. Horch, ”Selective Stimulation of Peripheral Nerve Fibers using Dual Intrafascicular Electrodes”, *IEEE Transactions on Biomedical Engineering*, vol. 40, No 5, 1993, pp 492-494.

[4] F. Rattay and M. Aberham, ”Modeling Axon Membranes for Functional Electrical Stimulation”, *IEEE Transactions on Biomedical Engineering*, vol. 40, No 12, 1993, pp 1201-1209.

[5] P.H. Veltink, J.A. Van Alste and H.B.K. Boom, ”Simulation of Intrafascicular and Extraneural Nerve Stimulation”, *IEEE Transactions on Biomedical Engineering*, vol. 35, No 1, 1988, pp 69-75.

[6] P.H. Veltink, J.A. Van Alste and H.B.K. Boom, ”A Modeling Study of Nerve Fascicle Stimulation”, *IEEE Transactions on Biomedical Engineering*, vol. 36, No 7, 1989, pp 683-692.

[7] K.E.I. Deurloo and Y. Holsheimer, ”Transverse tripolar stimulation for selective FNS”, *18th Conference IEEE Engineering in Medecine and Biology Society*, 1996, pp 331-332.

[8] A. Vuckovic, N.J.M. Rijkhoff and J.J. Struijk, ”Different Pulse Shapes to Obtain Small Fiber Selective Activation by Anodal Blocking: A Simulation Study”, *IEEE Transactions on Biomedical Engineering*, vol. 51, No 5, 2004, pp 698-706.

[9] N.J.M. Rijkhoff, J. Holsheimer, E.L. Koldewijn, J.J. Struijk, P.E.V. Van Kerrebroeck, F.M.J. Debruyne and H. Wijkstra, ”Selective Stimulation of Sacral Nerve Roots for Bladder Control: A Study by Computer Modeling”, *IEEE Transactions on Biomedical Engineering*, vol. 41, No 5, 1994, pp 413-423.

[10] J. Kybic, M. Clerc, T. Abboud, O. Faugeras, R. Keriven and T. Papadopoulo, ”A Common Formalism for the Integral Formulations of the Forward EEG Problem”, *IEEE Transactions on Biomedical Engineering*, vol. 24, No 1, 2005, pp 12-28.

[11] M. Clerc, J.-M. Badier, G. Adde, J. Kybic and T. Papadopoulo, ”Boundary element formulation for electricalal impedance tomography”, *ESAIM Proceedings*, Eric Cancès and Jean-Frédéric Gerbeau Editors, vol. 14, 2005, pp 63-71.

[12] D. Guiraud, T. Stieglitz, G. Taroni and J-L. Divoux, ”Original electronic design to perform epimysial and neural stimulation in paraplegia”, *Journal of Neural Engineering*, No 3, 2006, pp 276-286.

[13] M. Djilas, C. Azevedo, K. Yoshida and G. Cathbras ”Interpretation of ENG signal for FES closed-loop control”, *International Functional Electro-Stimulation Society (IFESS) Conference*, 2006.

[14] M. Djilas, K. Yoshida, M. Kurstjens and C. Azevedo-Coste ”Improving the signal-to-noise ratio on recordings with thin-film longitudinal intrafascicular electrodes using shielding cuffs”, *IEEE EMBS Conference on Neural Engineering*, 2007.