In this exercise, you will solve the stereo matching problem and the 1-D signal de-noising problem.

1 Stereo Matching Problem

Stereo matching is the problem of finding corresponding pixels, and their depth, in a left-right stereo image pair. These input images are rectified so that the scan lines are epi-polar lines i.e, the location of a pixel in the left and right image is shifted, compared to one another, depending on its distance from the cameras. This offset is called disparity. For each image pair, you will ultimately seek to output a disparity map for the left image indicating the disparity to the right image. The following figure illustrates the stereo matching problem. For each pixel the optimal ‘d’ is to be estimated where ‘d’ varies from \{0, 1, 2...d_{max}\}

![Stereo Correspondence](image)

Figure 1: Stereo Correspondence

The problem of recovering an accurate disparity map (\( \ell \)) can be posed as an energy minimization problem, where \( D_p(\ell_p) \), called data cost, is cost of assigning label \( \ell_p \); \( \mathcal{N} \) is a neighbourhood system on pixels and \( V_{pq}(\ell_p, \ell_q) \), called smoothness cost, is the cost of assigning labels \( \ell_p, \ell_q \) to adjacent pixels in the neighbourhood.

\[
\mathcal{E}(\ell) = \sum_{p \in I} D_p(\ell_p) + \lambda \sum_{(p, q) \in \mathcal{N}} V_{pq}(\ell_p, \ell_q)
\]

In this exercise, for the data cost you can use absolute intensity difference and a weighted truncated \( L_1 \) difference for the smoothness cost.

\[
D_p(d) = |I_L(x, y) - I_R(x + d, y)|
\]

\[
V_{pq}(\ell_p, \ell_q) = w_{pq} \times \min(|\ell_p - \ell_q|, K)
\]

To solve this optimization problem, you have to implement a simple algorithm called Iterated Conditional Modes (ICM). The ICM algorithm is one of the simplest methods for optimal decoding. In the ICM algorithm, we initialize the nodes to some starting state values and we then start cycling through the nodes in order. When we get to node i, we consider all states that node i could take, and replace its current state with the
state that minimizes the joint energy. We keep cycling through the nodes in order until we complete a full cycle without changing any nodes. At this point, we have reached a local optima of the joint energy function that can not be improved by changing the state of any single node.

For this exercise, we will use the Tsukuba image pair available here with 16 disparity levels, i.e, \( d = 0, 1, \ldots, 15 \). You can set \( \lambda \) and \( K \) to 20 and 2 respectively. The smoothness weight \( w_{pq} \) is set as below. You can evaluate the accuracy of your disparity map using the ground-truth available here.

\[
    w_{pq} = \begin{cases} 
    2 & |\ell_p - \ell_q| \leq 8 \\
    1 & \text{otherwise}
    \end{cases}
\]

2 1-D Signal Denoising

Given a noisy signal \( Y \), we want to recover a signal \( X \) that is both smooth and close to \( Y \). Let \( y = \{y_1, y_2, \ldots, y_N\} \) be the sum of smooth signal \( x = \{x_1, x_2, \ldots, x_N\} \) and gaussian noise \( \epsilon \).

\[
y = x + \epsilon
\]

The measure of closeness between two signals can be the sum of squares error.

\[
    E_{data}(x|y) = \frac{1}{2} \sum_{i=1}^{N} (y_i - x_i)^2
\]

A measure for the smoothness of the signal can be the following.

\[
    E_{smooth}(x) = \sum_{i=2}^{N} (x_i - x_{i-1})^2
\]

So, a smooth and close signal to a given noisy signal can be recovered by optimizing the following function.

\[
    \arg\min E_{data}(x|y) + \lambda E_{smooth}(x)
\]

You can optimize this function using the ICM algorithm. For this exercise, you are provided with two Matlab functions (\texttt{func1.m} and \texttt{func2.m}) which generate noisy signals. You have to denoise and report the mean square error between the recovered signal and the original noisy signal, i.e, between \( x \) and \( y \).

3 Implementation

To keep the runtime in reasonable limits, implement the stereo matching problem in C++. For loading and saving images you can use the Imagine++ library. An example program is available here. However, for the signal denoising problem you can use Matlab.

4 Submission

Mail your source code and report to nikos.komodakis@enpc.fr The report should include the obtained disparity map and accuracy for the stereo matching problem. For the signal denoising problem include plots of the original noisy signal (in black) and the recovered signal (in red). Also generate plots for different values of \( \lambda \) and report what do you observe. The deadline for this exercise is 24th December.