Vision 3D artificielle - Final exam (duration: 1:30h)

P. Monasse and R. Marlet

Novembre 15, 2011

You can choose to answer in French or English, at your convenience.

1 Computation of homography

In the computation of a homography from (finite) point matches, we can write the vector equation $X_i' \times (HX_i) = 0$ as two independent linear equations in the parameters $h = \begin{pmatrix} H_{11} & H_{12} & \cdots & H_{33} \end{pmatrix}$ of H:

$$\begin{pmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i' \end{pmatrix} h = 0.$$

From 4 point matches, we get an 8×9 system in h, so h is a non-null vector of the kernel of the stacked matrix A. There seems to be always a unique solution but obviously there are degenerate situations. The following questions examine these degeneracies.

1. Explain why there is no solution if $X_1 = X_2$ and points X_1' and X_2' are distinct.
2. By examining the structure of the matrix A, explain why there is no unique solution if the points X_1, X_2 and X_3 are distinct but aligned.
3. Show the form of the homography matrix H keeping fixed all points of line $y = 0$ and also the point $(0, 1)$ of Euclidean plane. Explain why it is a 1-parameter family of homographies.

2 Sampson error for fundamental matrix

We recall that the Sampson error $\epsilon_S(X)$ for a given geometric model, an approximation of the geometric error based on the algebraic error $\epsilon(X)$, can be written:

$$\epsilon_S = \epsilon^T (JJ^T)^{-1} \epsilon$$

with J the matrix of partial derivatives (Jacobian matrix) of ϵ as a function of data point X. This exercise computes the formula of Sampson error for fundamental matrix F.

The algebraic error is $P'^T F P = 0$ with $P = \begin{pmatrix} x & y & 1 \end{pmatrix}^T$ and $P' = \begin{pmatrix} x' & y' & 1 \end{pmatrix}^T$.

1. Compute J at point $X = \begin{pmatrix} x & y & x' & y' \end{pmatrix}$.
2. Write formula for Sampson error associated to F.
3. Discuss its relation to geometric error.

3 Recovery of projection matrices from F

From P and P', 3×4 projection matrices in left and right images, we can compute F, the fundamental matrix. The goal of this exercise is to recover all possible pairs of matrices (P, P') from a given F. We note e' the epipole in right image ($F^T e' = 0$).

1. Show that if $P = (I|0)$ and $P' = ([e']_\times F | e')$, then $S = P'^T F P$ is skew-symmetric ($X^T S X = 0$ for all X).
2. Deduce that the above pair (P, P') gives rise to F. Why is the right camera of this particular solution not a pinhole camera?
3. Show that if $P = (I|0)$ and $P' = (A|a)$ (with A a 3×3 matrix and a a 3-vector), then $F = [a]_\times A$.
4. Suppose $F = [a]_\times A = [a']_\times A'$. Prove that there is some $\lambda \neq 0$ and some 3-vector v such that

$$a' = \lambda a \text{ and } \lambda A' = A + a v^T.$$

5. Prove that if $P = (I|0)$, then $P' = ([e']_\times F + e' v^T | \lambda e')$ with $\lambda \neq 0$ and v a 3-vector.
6. Explain why if the pair (P, P') yields F then so does $(PH, P'H)$ for any 4×4 matrix H, then show that the general solution is such that $P = (P_{3\times 3} | P_4)$ with $P_{3\times 3}$ invertible, and

$$P' = ([e']_\times F + e' v^T) P + e' w^T$$

with w a 4-vector such that
$$\begin{pmatrix} P \\ w^T \end{pmatrix}$$
be invertible.

4 Feature detection and description

1. Between a 90° corner and a 45° corner, which one has the strongest response according to the Harris-Stephens criterion, and why?

   To answer this question, you may simply compare the case of the central point in images I and I' defined by:

   $I =$

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	0	0	0
0	0	0	0	0

   $I' =$

0	0	0	0	1
0	0	0	1	1
0	0	1	1	1
0	0	0	0	0
0	0	0	0	0

   As indicated in the lecture on the Harris corner detector, directional derivatives can be computed convoluting respectively with $[-1\ 0\ 1]$ and $[-1\ 0\ 1]^T$.

   You may restrict yourself to a *derivative* Gaussian, and forget about the *integration* Gaussian. To simplify calculations, you may also use an approximate Gaussian kernel defined as:

   $g = \frac{1}{16}$

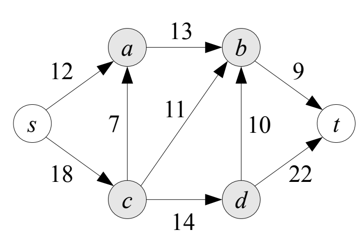
1	2	1
2	4	2
1	2	1

2. Does the answer differ if you consider the corner strength as defined by Brown et al. (2005), i.e., the harmonic mean of the eigenvalues?

3. Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ be detected SIFT points in image I, $\mathbf{x}'_1$ be a detected SIFT points in image I' and $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}'_1$ be the corresponding SIFT descriptors. Assume that $\|\mathbf{v}_1 - \mathbf{v}_2\| = 0.1$ and $\|\mathbf{v}_1 - \mathbf{v}_3\| = 0.11$. What can be said of a possible match between $\mathbf{x}_1$ and $\mathbf{x}'_1$?

5 Graph cuts

1. What is the minimal cut of the following graph, and why?



2. When using a graph cut for estimating a disparity map as explained in the course, suppose you use a zero-mean normalized sum of square differences (E_{ZNSSD}) rather than a zero-mean normalized cross correlation (E_{ZNCC}). Has anything to be changed in edge weight definition?