3D Computer Vision - MVA final exam
(duration: 2h)

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The exercises are independent. You can choose to answer in French or English, at your convenience.

1 Regularizers

Consider the parameterized functions f_α: $x \to |x|^\alpha + |1-x|^\alpha$.

1. Find the minimum and its argument(s) of the function f_α for $\alpha > 1$.
2. Find the minimum and its argument(s) of the function f_α for $\alpha = 1$.
3. Find the minimum and its argument(s) of the function f_α for $0 < \alpha < 1$.
4. Find the minimum and its argument(s):

$$\min_{\{x_0, x_1, \dots, x_{n+1}\}} \sum_{i=0}^{n} (x_i - x_{i+1})^2 \tag{1}$$

   under the constraints $x_0 = 0$ and $x_{n+1} = 1$.

5. Let f be a function defined on $[0, +\infty)$ with $f(0) = 0$. Show that for $n > 0$

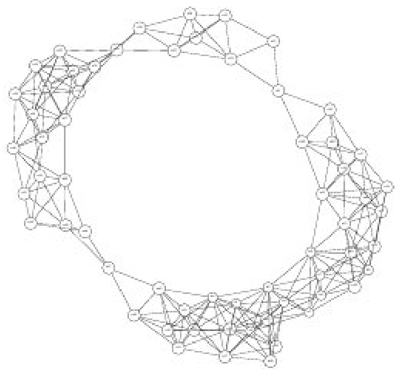
$$f(1) \geq n\, f(\frac{1}{n}) \text{ if } f \text{ is convex,} \qquad f(1) \leq n\, f(\frac{1}{n}) \text{ if } f \text{ is concave.} \tag{2}$$

6. Deduce some consequences about the effects of different regularizers in graph cuts applications, such as in computing a disparity map.

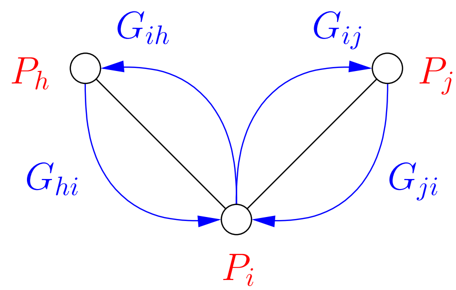
2 Solvable Viewing Graph

The viewing graph is formed by the different (uncalibrated) cameras of a multi-view stereo acquisition as vertices, and edges between two vertices if their corresponding views have sufficient matchings to compute the fundamental matrix. It is said *solvable* if fixing one of the 3×4 projection matrices to $P_0 = \begin{pmatrix} I_3 & 0 \end{pmatrix}$ automatically determines all other projection matrices.

7. Justify that we can always assume in a multi-view stereo setup of uncalibrated cameras, without loss of generality, that one projection matrix is P_0.
8. In a stereo pair, given P_0 and P, write the expression of the fundamental matrix.



A viewing graph



Question 16

9. Show that if $G \in \mathbb{R}^{4\times 4}$ is invertible, the fundamental matrix associated to $P_1 G$ and $P_2 G$ is the same as with P_1 and P_2.

10. Show that the graph of a stereo pair with one edge is not solvable. Exhibit a simple example.

11. Show that the homogeneous coordinates of the center $c(P)$ of a camera can be deduced from its projection matrix P. Propose a procedure to compute them.

12. Show that a complete graph of three views is not solvable if the three camera centers are aligned. Therefore, we amend the definition to say the graph is solvable if for almost all positions of camera centers the property is satisfied.

13. Show that a solvable viewing graph is biconnected: removing any single edge preserves the connectedness of the graph.

14. Show that $G \in \mathbb{R}^{4\times 4}$ invertible satisfies $PG = aP$ with $a \neq 0 \in \mathbb{R}$ if and only if there is some $v \in \mathbb{R}^4$ satisfying

$$G = aI_4 + c(P)\, v^\top. \tag{3}$$

15. Show that the right hand side of (3) is invertible if and only if $a + c(P)^\top v \neq 0$ and write then the expression of G^{-1}.
For vertices i and j linked in the graph, we associate $G_{ij} \in \mathbb{R}^{4\times 4}$ invertible, with $G_{ji} = G_{ij}^{-1}$. It is understood that G_{ij} is applied to cameras P_i and P_j, which does not change their fundamental matrix F_{ij} according to Question 9.

16. Suppose we have edges (h,i) and (i,j). Show that $G_{hi}G_{ij}^{-1}$ can be written as

$$G_{hi}G_{ij}^{-1} = a_{hij} I_4 + c(P_i)\, v_{hij}^\top. \tag{4}$$

17. Show that the graph is solvable if and only if (4) is satisfied at each vertex of degree 2 or more and there is an invertible matrix $H \in \mathbb{R}^{4\times 4}$ such that

$$\forall i, j, \exists s_{ij} \in \mathbb{R} \neq 0 \text{ such that } G_{ij} = s_{ij} H. \tag{5}$$

18. Show that the graph is solvable if and only if $v_{hij} = 0$ for all edge pairs (h,i) and (i,j).

19. Show that we can check whether a graph is solvable by solving a system of rational equations (ratio of polynomials). What do you think of the feasability?