3D Computer Vision - MVA final exam
(duration: 2h)

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The exercises are independent. You can choose to answer in French or English, at your convenience.

1 Camera Center Coordinates

1. Let A a linear form on $\mathbb{R}^n$. Show that there is a unique $x(A) \in \mathbb{R}^n$ such that $AX = x(A)^\top X$ for all X.
2. Let $P = \begin{pmatrix} P_1^\top \\ P_2^\top \\ P_3^\top \end{pmatrix}$ the 3×4 projection matrix of a camera ($P_i \in \mathbb{R}^4$)and $C \in \mathbb{R}^4$ the homogeneous coordinates of its center. Show that $P_i^\top C = 0$ for $i = 1, 2, 3$.
3. Consider the linear form A on $\mathbb{R}^4$ defined by
$$AX = \left|\begin{pmatrix} P_1 & P_2 & P_3 & X \end{pmatrix}\right|. \tag{1}$$
Show that $P_i^\top x(A) = 0$.
4. Using a development of the determinant along the last column, write an expression of the coordinates of C involving determinants of matrices extracted from P.

2 Multi-View Stereo

We recall from the course that the existence of one 3D point X viewed in 3 cameras as x_1, x_2, x_3 depends on the rank of the 6×2 matrix
$$M = \begin{pmatrix} [x_2]_\times K_2 R_2 K_1^{-1} x_1 & [x_2]_\times K_2 T_2 \\ [x_3]_\times K_3 R_3 K_1^{-1} x_1 & [x_3]_\times K_3 T_3 \end{pmatrix} \tag{2}$$
with $x_i = \lambda_i K_i(R_i X + T_i)$, $R_1 = I_3$ and $T_1 = 0$. We have $x_i, X, T_i \in \mathbb{R}^3$, $K_i \in \mathbb{R}^{3\times3}$, $R_i \in SO(3)$.

5. Remind the number of solutions X as a function of the rank of M.
6. Show the result (written but not proved in the course) that $M = 0$ if and only if the three 3D camera centers C_i and X are aligned.
7. Explain geometrically why, provided the existence of X, we can expect in general that given x_2 and x_3 (and all products $K_i R_i K_1^{-1}$ and $K_i T_i$) we can recover x_1.
8. Prove by computation that the exception to the previous question happens if and only if points C_i and X are coplanar.

3 Graph Cuts Meet RANSAC

9. Write as pseudo-code the RANSAC algorithm applied to the estimation of a homography H from pairs of hypothetically matching image points (x_i, x'_i).

10. The variant LO-RANSAC proposes as "local optimization" to refine a hypothetic H by recomputing a new H' from all inliers of H. Explain why the inliers of H' may be different from the ones of H.

11. Propose an iterative refinement of H in LO-RANSAC.

12. Given a hypothesis for H, we use as error metrics the distance from points Hx_i and x'_i. Depending on an inlier/outlier threshold δ on such distance, show that the labelling L of point pairs (0 for inlier, 1 for outlier) in RANSAC minimizes some energy $E_d(L)$.

13. In practice, it is observed that close pairs (in $\mathbb{R}^4$) (x_i, x'_i) and (x_j, x'_j) should usually have the same label $L_i = L_j$. Propose an energy $E_r(L)$ whose lower values promote this property.

14. Show that the optimal labelling minimizing the energy $E(L) = E_d(L) + \lambda E_r(L)$ can be found via graph-cuts (explain the contruction of the graph).

15. How can we mix the ideas of the graph cuts and of LO-RANSAC? This was done in the literature under the name GC-RANSAC.