3D Computer Vision - MVA final exam
(duration: 2h)

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The exercises are independent. You can choose to answer in French or English, at your convenience.

1 Projection of matrices

1. The Frobenius norm in $\mathbb{R}^{m\times n}$ is defined as $\|A\|_F^2 = \operatorname{trace}(A^\top A)$. Write its expression as a function of coefficients A_{ij} and justify it is a matrix norm.
2. Propose a simple algorithm based on SVD to get the projection of a matrix A (with respect to the Frobenius norm) on the set of matrices of rank less than $\min(m,n)$. Prove the validity of the algorithm.
3. In which part of the stereovision pipeline is it useful?

2 Essential matrix

4. Recall a necessary and sufficient condition based on SVD for a matrix $E \in \mathbb{R}^{3\times 3}$ to be the essential matrix of a stereo pair.
   We will prove that the condition can also be rewritten without involving the SVD:
   $$2EE^\top E = \operatorname{trace}(EE^\top)E. \tag{1}$$
5. Show that (1) is a necessary condition.
6. Show that E satisfying (1) cannot be invertible.
7. Find a trivial solution for this equation. We exclude it in the next question.
8. Considering the SVD of E, show that (1) is a sufficient condition.

3 Graph cuts

9. Recall what are the global moves (α,β) swap and α-expansion, with $\alpha,\beta \in \mathcal{L}$, the set of labels of nodes in a graph cut problem.
10. Given a set of label assignments $X = \{x_i\}$ for the nodes, show that a necessary and sufficient condition for computing the *optimal* (α,β) swap $X' = \{x'_i\}$ of X as a binary graph-cut problem is
    $$V_{ij}(\alpha,\alpha) + V_{ij}(\beta,\beta) \le V_{ij}(\alpha,\beta) + V_{ij}(\beta,\alpha), \tag{2}$$
    noting $V_{ij} \ge 0$ the pairwise potential between nodes i and j.

11. Show that a necessary and sufficient condition for computing the *optimal* α-expansion X' of X as a binary graph-cut problem is
$$V_{ij}(\alpha,\alpha) - V_{ij}(\beta,\alpha) \le \min_{\gamma\in\mathbb{L}} \left(V_{ij}(\alpha,\gamma) - V_{ij}(\beta,\gamma)\right). \tag{3}$$

12. Show that if $V_{ij}(x,y)$ is a distance on $\mathcal{L}$, then (3) is satisfied.

4 Trifocal tensor and epipoles

13. Suppose we have three cameras in space of projection matrices P_1, P_2 and P_3. Show that without loss of generality, we can assume in the following that $P_1 = \begin{pmatrix} Id_3 & 0 \end{pmatrix}$.

14. We write $P_2 = \begin{pmatrix} A & a_4 \end{pmatrix}$ and $P_3 = \begin{pmatrix} B & b_4 \end{pmatrix}$. Show that the fundamental matrix between camera 2 and 1 is $[a_4]_\times A$ and between 3 and 1 is $[b_4]_\times B$.

15. Compute the fundamental matrix between cameras 3 and 2.

16. Show that the center of the first camera is projected into a_4 and b_4 in views 2 and 3.

17. Using the trifocal constraints as written in the course slides, show that the 3 slices of the trifocal tensor are
$$T_i = a_i b_4^\top - a_4 b_i^\top. \tag{4}$$
with a_i and b_i ($i \in \{1,2,3\}$) the columns of A and B.

18. Show that the epipoles of image 1 in image 2 can be obtained as the commmon intersection of the lines represented by the left null-vectors of T_1, T_2, T_3.

19. How do we deduce the epipole of image 1 in image 3 from T_1, T_2, T_3?