1 Camera Center Coordinates

1. Let $A$ a linear form on $\mathbb{R}^n$. Show that there is a unique $x(A) \in \mathbb{R}^n$ such that $AX = x(A)^{\top}X$ for all $X$.

2. Let $P = \begin{pmatrix} P_1^\top \\ P_2^\top \\ P_3^\top \end{pmatrix}$ the $3 \times 4$ projection matrix of a camera ($P_i \in \mathbb{R}^4$) and $C \in \mathbb{R}^4$ the homogeneous coordinates of its center. Show that $P_i^\top C = 0$ for $i = 1, 2, 3$.

3. Consider the linear form $A$ on $\mathbb{R}^4$ defined by $AX = \begin{vmatrix} x_2^2 & K_2R_2K_1^{-1}x_1 \\ x_3^2 & K_3R_3K_1^{-1}x_1 \\ x_3^2 & K_3T_3 \end{vmatrix}$. (1)

Show that $P_i^\top x(A) = 0$.

4. Using a development of the determinant along the last column, write an expression of the coordinates of $C$ involving determinants of matrices extracted from $P$.

2 Multi-View Stereo

We recall from the course that the existence of one 3D point $X$ viewed in 3 cameras as $x_1, x_2, x_3$ depends on the rank of the $6 \times 2$ matrix $M = \begin{pmatrix} [x_2^2] & K_2R_2K_1^{-1}x_1 \\ [x_3^2] & K_3R_3K_1^{-1}x_1 \\ [x_3^2] & K_3T_3 \end{pmatrix}$ (2)

with $x_i = \lambda_iK_i(R_iX + T_i)$, $R_1 = I_3$ and $T_1 = 0$. We have $x_i, X, T_i \in \mathbb{R}^3$, $K_i \in \mathbb{R}^{3 \times 3}$, $R_i \in SO(3)$.

5. Remind the number of solutions $X$ as a function of the rank of $M$.

6. Show the result (written but not proved in the course) that $M = 0$ if and only if the three 3D camera centers $C_i$ and $X$ are aligned.

7. Explain geometrically why, provided the existence of $X$, we can expect in general that given $x_2$ and $x_3$ (and all products $K_iR_iK_1^{-1}$ and $K_iT_i$) we can recover $x_1$.

8. Prove by computation that the exception to the previous question happens if and only if points $C_i$ and $X$ are coplanar.
3 Graph Cuts Meet RANSAC

9. Write as pseudo-code the RANSAC algorithm applied to the estimation of a homography $H$ from pairs of hypothetically matching image points $(x_i, x'_i)$.

10. The variant LO-RANSAC proposes as “local optimization” to refine a hypothetic $H$ by recomputing a new $H'$ from all inliers of $H$. Explain why the inliers of $H'$ may be different from the ones of $H$.

11. Propose an iterative refinement of $H$ in LO-RANSAC.

12. Given a hypothesis for $H$, we use as error metrics the distance from points $Hx_i$ and $x'_i$. Depending on an inlier/outlier threshold $\delta$ on such distance, show that the labelling $L$ of point pairs (0 for inlier, 1 for outlier) in RANSAC minimizes some energy $E_d(L)$.

13. In practice, it is observed that close pairs (in $\mathbb{R}^4$) $(x_i, x'_i)$ and $(x_j, x'_j)$ should usually have the same label $L_i = L_j$. Propose an energy $E_r(L)$ whose lower values promote this property.

14. Show that the optimal labelling minimizing the energy $E(L) = E_d(L) + \lambda E_r(L)$ can be found via graph-cuts (explain the construction of the graph).

15. How can we mix the ideas of the graph cuts and of LO-RANSAC? This was done in the literature under the name GC-RANSAC.