

# Piecewise-Planar 3D Reconstruction with Edge and Corner Regularization



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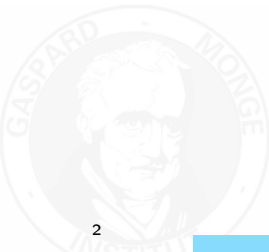
SGP 2014

# Need

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3D models of existing buildings:

- thermal or acoustic simulations
- light and shadow casting
- Building Information Models

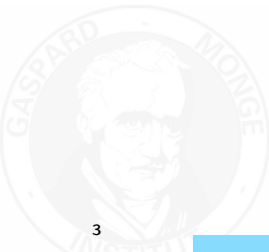


# Existing solutions

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Laser point clouds + semi-automatic surface reconstruction

- error prone
- time consuming
- expensive

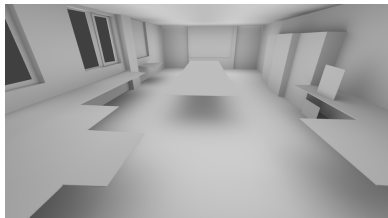
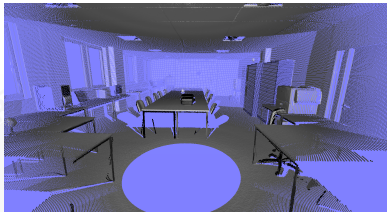


# Objective

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Automatic 3D surface reconstruction from point cloud

- watertight without self-intersection
- extends in a plausible manner in hidden regions
- piecewise planar

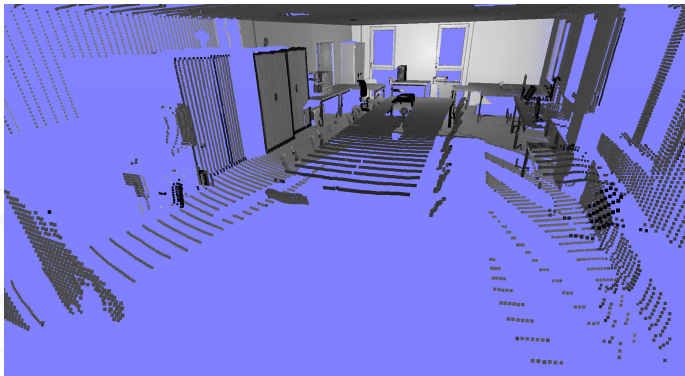


# Challenges

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Two main challenges

- ubiquitous occlusions
- sampling anisotropy



## Limitation of existing methods

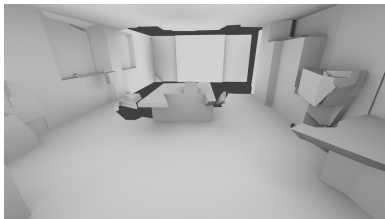
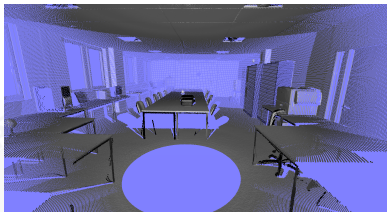
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- Smooth surfaces priors are inadequate
- Intersects only pairs of planes that are adjacent in range image
- Manhattan world assumption: too restrictive
- Watertight solutions not guaranteed
- Voxelization: biased, expensive
- Delaunay tetrahedralization: visible regions only

- Plane arrangement
  - Planes detected in the point cloud using region growing
  - Hidden planes hypotheses (ghosts) guessed from the edges of detected polygons
- binary labelization of the 3D space
  - pairwise MRF (2nd order factors)
  - solution with graph-cut
- Advantages
  - watertight solution
  - primitives can expand far beyond their visibility area
  - allows the use of hidden planes hypotheses
  - sharp surface reconstruction

Limitations:

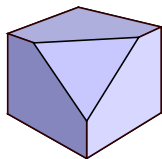
- anisotropy of laser point clouds is a problem
- missing plane hypotheses
- surface area minimization creates holes and cutted corners



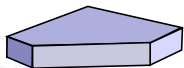
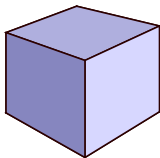


# Surface area vs Edges length vs Corners count

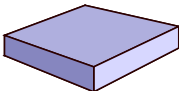
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→  
larger surface area  
shorter edges  
fewer corners



→  
larger surface area  
longer edges  
fewer corners



# Our approach

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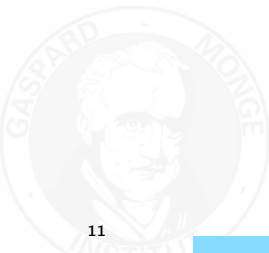
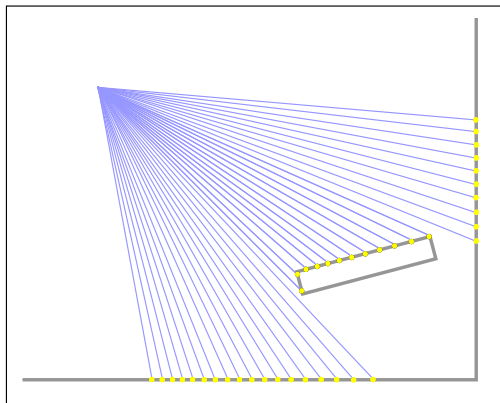
## Contributions

- treatment of sampling anisotropy
- better and new plane hypotheses
- higher-order regularization:
  - **length of edges** (4th order factors)
  - **number of corners** (8th order factors)
- globally near optimal solutions using LP relaxation

# Method overview

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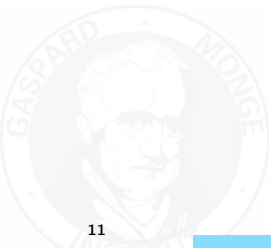
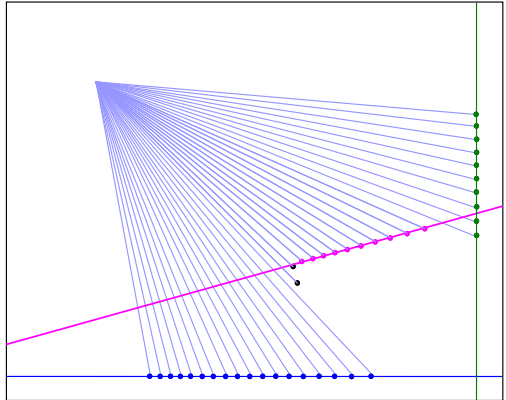
- laser measures



# Method overview

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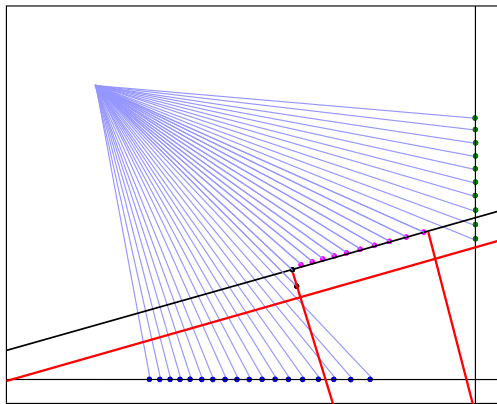
- laser measures
- planes detection



# Method overview

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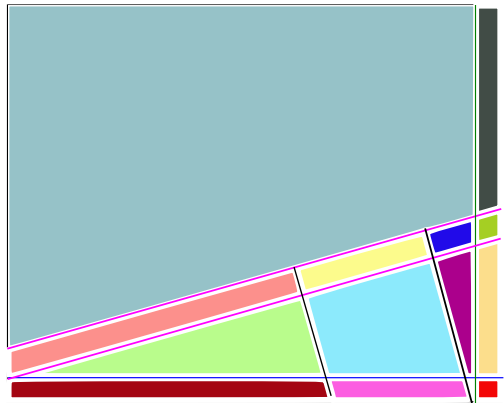
- laser measures
- planes detection
- region polygonization and Ghosts creation



# Method overview

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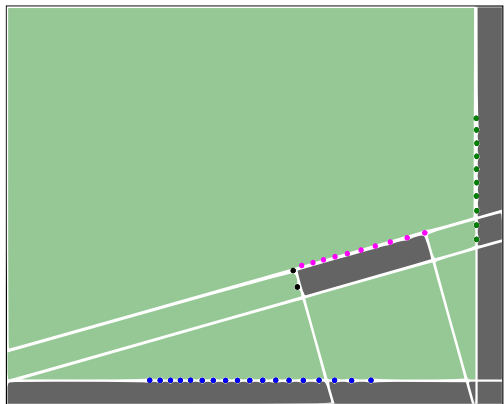
- laser measures
- planes detection
- region polygonization and Ghosts creation
- volume partition using a plane arrangement



# Method overview

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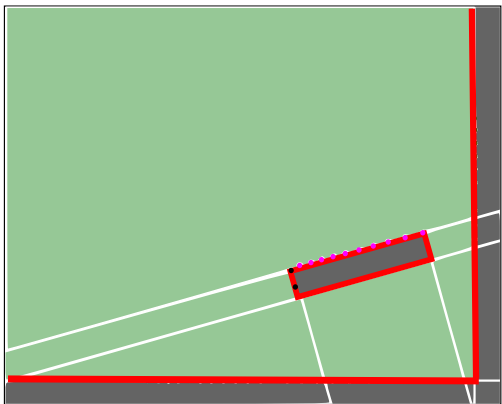
- laser measures
- planes detection
- region polygonization and Ghosts creation
- volume partition using a plane arrangement
- LP Binary labelization



# Method overview

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- laser measures
- planes detection
- region polygonization and Ghosts creation
- volume partition using a plane arrangement
- LP Binary labelization
- surface extraction





# Plan detection

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Region growing approach.

- compute point normals with a method that preserves sharp features [Boulch et al. 2012]
- locally planar region as seeds
- grow region from seeds
- keep plane equations updated using online least-square fitting



# Plan detection

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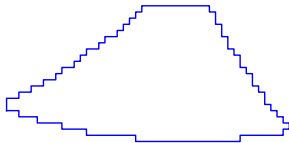
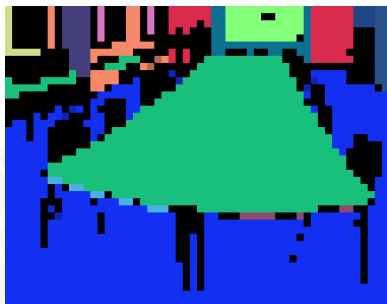
plane fusion to recover from over-segmentation using robust statistical criteria [Boulch et al. 2014]



# Polygonization

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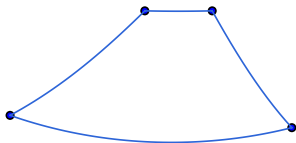
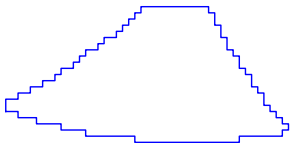
- Extraction of the the boundary pixel chain



# Polygonization

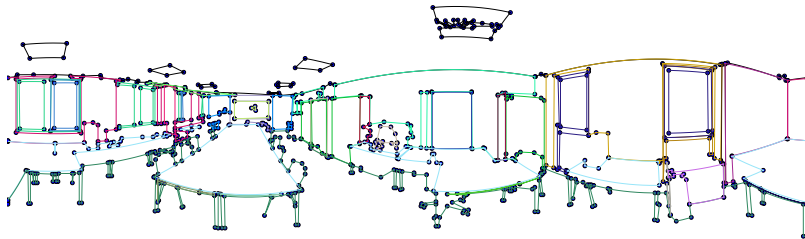
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- Extraction of the the boundary pixel chain
- Polygon simplification by greedy merging of adjacent edges, keeping maximum distance to the original polygon below 2 pixels (aliasing)



# Polygonization

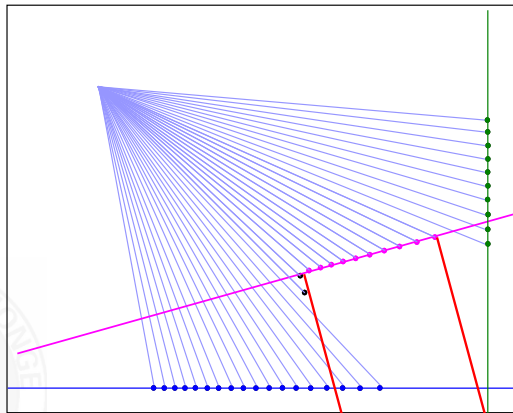
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# Orthogonal ghosts

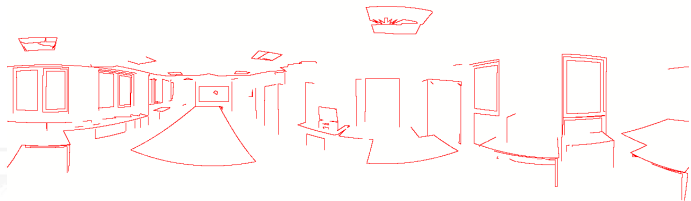
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We generate an orthogonal half-plane for each polygon edge



# Orthogonal ghosts

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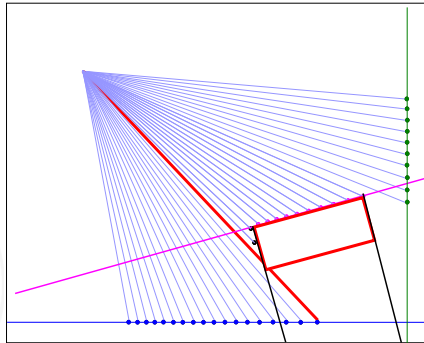


# Parallel ghosts

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thin objects (tables, screens etc.)

- not enough points on the side
- find largest valid thickness



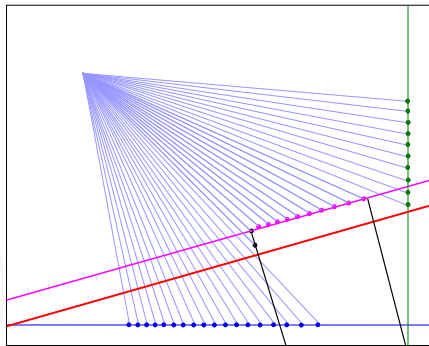


# Parallel ghosts

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thin objects (tables, screens etc.)

- not enough points on the side
- find largest valid thickness
- create parallel ghost



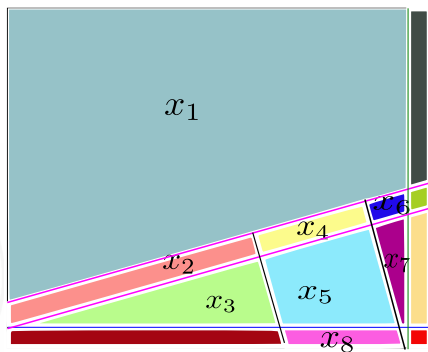
# Surface reconstruction

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Once we have all the plane candidates ,

- partition the volume with a plane arrangement
- label each cell as empty or full

$$x = (x_1, \dots, x_N) \in \{0, 1\}^N$$



# Surface reconstruction

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$$E(x) = E_{data}(x) + E_{regul}(x)$$

labelization through minimization of a sum of terms

- data terms

$$E_{data}(x) = E_{prim}(x) + E_{vis}(x)$$

- the regularization terms

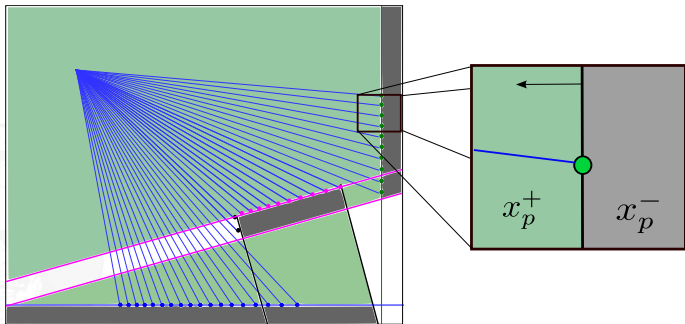
$$E_{regul}(x) = E_{area}(x) + E_{edge}(x) + E_{corner}(x)$$

- allows to cope with noisy measurement
- allows completion in hidden regions

## Data term

- cells in front of labeled points should be empty
- cells just behind the points should be full

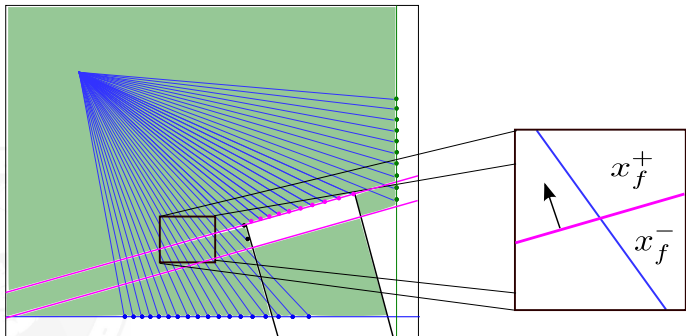
$$E_{\text{prim}}(\mathbf{x}) = \sum_{p \in \mathcal{P}} w_p^{\text{aniso}} (x_p^+ + (1 - x_p^-)) \quad (1)$$



## Data term

Facets on the surface should not intersect rays

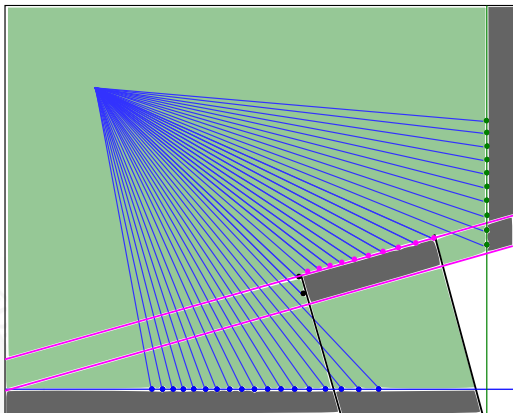
$$E_{\text{vis}}(\mathbf{x}) = \sum_{\substack{p \in \mathcal{P}, f \in \mathcal{F} \\ \omega_p \cap f \neq \emptyset}} w_p^{\text{aniso}} |x_{f^+} - x_{f^-}| \quad (2)$$



# Data term

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Data term are not enough to label all cells



# Surface area regularization

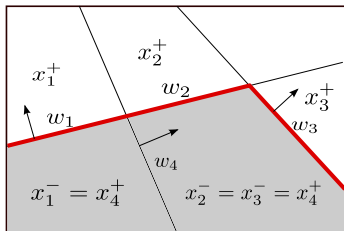
- the total area of the surface is

$$E_{\text{area}}(\mathbf{x}) = \sum_{f \in \mathcal{F}} w_f |x_{f+} - x_{f-}| \quad (3)$$

with

$$w_f = a_f / \sigma^2$$

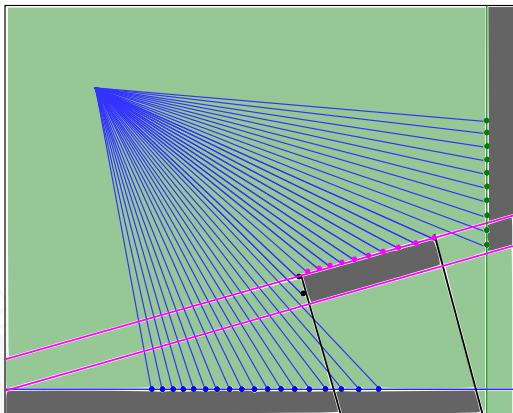
where  $\sigma$  is a scale parameter and  $a_f$  the area of the facet



## Data term + Area term

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Area term does not fill large gaps

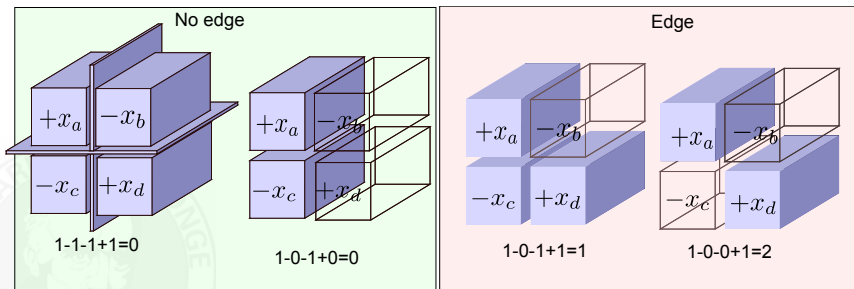




# Edge length regularization

existence of an edge as a linear function of the adjacent cell binary values:

$$h_e(x) = x_a - x_b - x_c + x_d$$



## Edge length regularization

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The total edge length of the surface is penalized in the optimized energy using

$$E_{\text{edge}}(\mathbf{x}) = \sum_{e \in \mathcal{E}} w_e |h_e(\mathbf{x})| \quad (4)$$

With

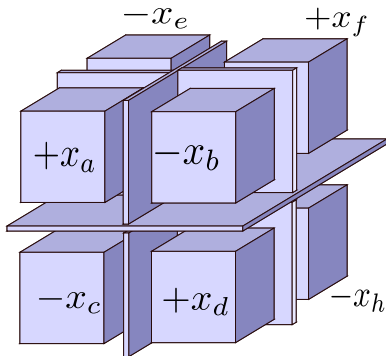
$$w_e = \frac{l_e}{\sigma} w_{\text{ang}}(\alpha_e) \quad (5)$$

with  $\sigma$  the scale parameter and  $w_{\text{ang}}(\alpha_e)$  a function of the angle between the two planes

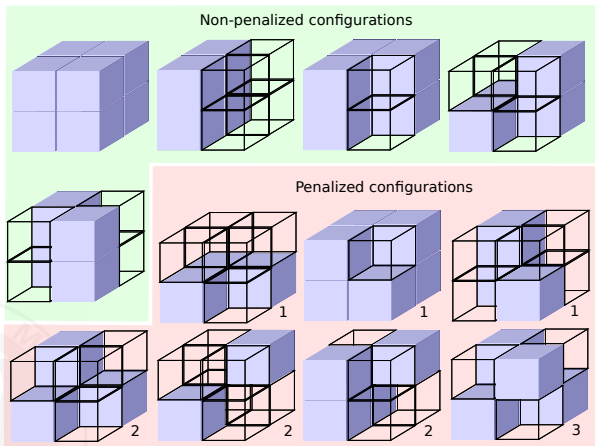
## Corners count regularization

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$$h_v(x) = x_a - x_b - x_c + x_d - x_e + x_f + x_g - x_h$$



# Corners count regularization



## Corners count regularization

---

We penalize the number of corner in the reconstructed surface by adding to the minimized energy the term

$$E_{\text{corner}}(\mathbf{x}) = \sum_{v \in \mathcal{V}} w_v |h_v(\mathbf{x})| \quad (6)$$

$w_v$  depends on the three angles between each pair of plane:

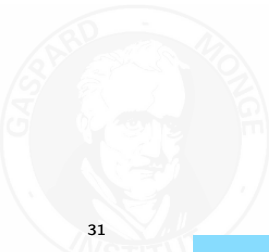
$$w_v = w_{\text{ang}}(\alpha_1, \alpha_2, \alpha_3) \quad (7)$$

The corner count terms correspond to potentials of order up to 8 in the context of MRFs

# Optimization

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- 8th order potential are challenging for MRF minimization methods.
  - Tree-reweighted Belief Propagation, extremely slow to converge
  - Lazy Flipper : local minimum, extremely suboptimal
- We formulate the labeling problem as a Mixed-integer programming problem



## Optimization

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The total minimized energy can be written as

$$E(x) = \zeta + \sum_i w_i |H_i \cdot x|$$

with  $\zeta$  a constant and each  $H_i$  is a sparse vector using an auxiliary variable  $y_i$ , each term can be formulated as linear term with additional constraints

$$w_i |H_i \cdot x| = \min_{y_i} w_i y_i \quad \text{s.t.} \quad -y_i \leq H_i \cdot x \leq y_i \quad (8)$$

thus we aim to solve the integer program

$$\min_{x,y} \sum_i w_i y_i \quad \text{s.t.} \quad x \in \{0, 1\}^N, \forall i : -y_i \leq H_i \cdot x \leq y_i \quad (9)$$

# Optimization

---

we aim to solve the integer program

$$\min_{x,y} \sum_i w_i y_i \quad \text{s.t.} \quad x \in \{0,1\}^N, \forall i : -y_i \leq H_i \cdot x \leq y_i \quad (10)$$

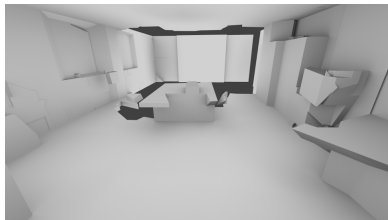
we relax the integer constraint  $x \in \{0,1\}^N$  to the box constraint  $x \in [0,1]^N$ :

$$\min_{x,y} \sum_i w_i y_i \quad \text{s.t.} \quad x \in [0,1]^N, \forall i : -y_i \leq H_i \cdot x \leq y_i \quad (11)$$

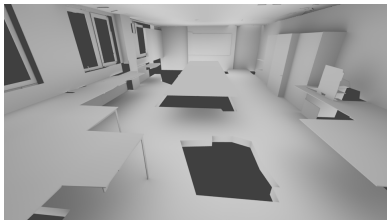
This is a standard Linear Program, We solve it using the dual simplex in the commercial Mosek© solver. After rounding to solution to integers we obtained an increase of energy not greater than 8%.



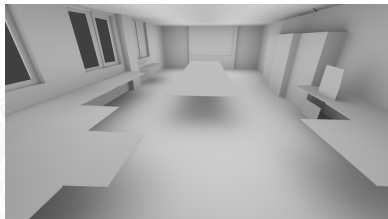
# Results



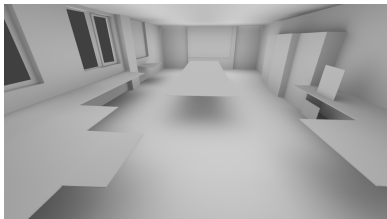
chauve & al



surface only



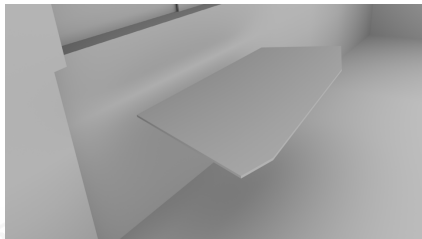
edges only



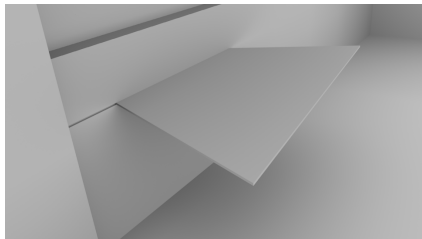
corners only

# Results

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edges



corners

# Results

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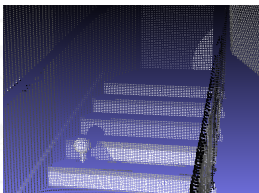
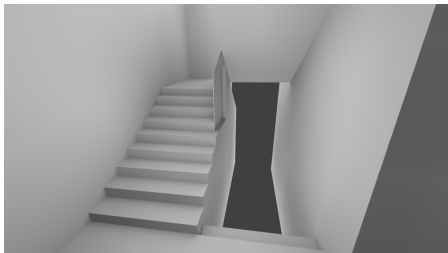
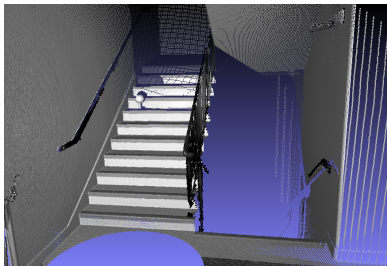
corners



edge+corners

# Results

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point cloud

reconstruction

# Conclusion

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- allows plausible completion in hidden regions
- handles anisotropy
- edge and corner regularization superior to area term for completion
- near-optimal global solution using efficient LP relaxation

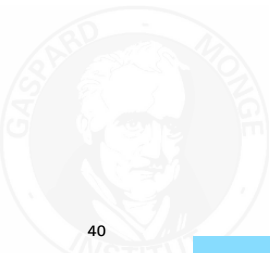
## Futur work

- photogrammetry
- better scalability to large scenes

# Links

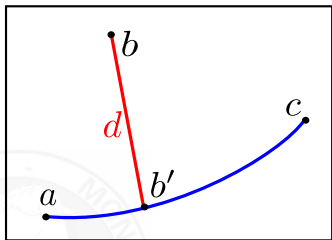
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- This work was partly supported by Bouygues Construction interested in automatic BIM generation from existing buildings
- emails:
  - [martin.de-la-gorce@enpc.fr](mailto:martin.de-la-gorce@enpc.fr)
  - [alexandre.boulch@enpc.fr](mailto:alexandre.boulch@enpc.fr)
  - [renaud.marlet@enpc.fr](mailto:renaud.marlet@enpc.fr)
- IMAGINE team website:
  - [imagine.enpc.fr](http://imagine.enpc.fr)

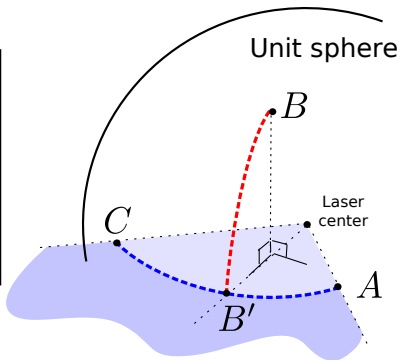


# Polygonization

The polygons have curvy edges in the image coordinate system. we compute the distance of a point to a curvy segment using geodesic projection in the the sphere



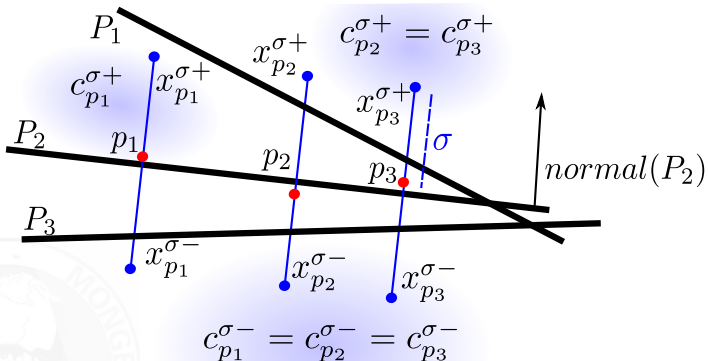
Laser image





## Data term

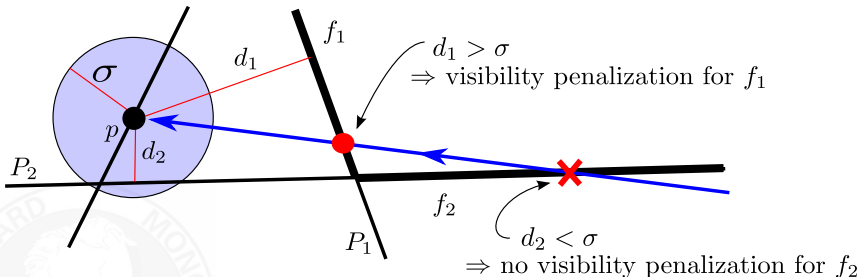
The surface should pass near observed points:



## Data term

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The surface should not intersect any segment joining the scanner center and the observed points



## Data term

---

- We penalize full cell just in front of a point and empty cell just behind a point

$$E_{\text{prim}}(\mathbf{x}) = \sum_{p \in \mathcal{P}} w_p^{\text{aniso}}(P_p) (x_p^{\sigma^+} + (1 - x_p^{\sigma^-})) \quad (12)$$

- We use a penalization weight that take anisotropy into account

$$w_p^{\text{aniso}}(P) = \frac{d^2}{\sigma^2} \Delta_\theta \Delta_\phi \frac{\sin \phi}{\cos \psi} \quad (13)$$

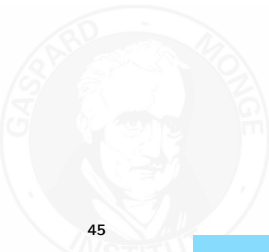
with  $\theta$  the azimuth angle,  $\phi$  the polar angle,  $\Delta_\theta$  and  $\Delta_\phi$  the two steps of the scan.

## Data term

---

We penalize the use of facets that intersect the segments joining the laser center and the observed points

$$E_{\text{vis}}(\mathbf{x}) = \sum_{\substack{p \in \mathcal{P}, f \in \mathcal{F} \\ \omega_p \cap f \neq \emptyset, d(p, P_f) \leq \sigma}} w_p^{\text{aniso}}(P_f) |x_{f+} - x_{f-}| \quad (14)$$



## Corners count regularization

---

We penalize the number of corner in the reconstructed surface by adding to the minimized energy the term

$$E_{\text{corner}}(\mathbf{x}) = \sum_{v \in \mathcal{V}} w_v |h_v(\mathbf{x})| \quad (15)$$

$w_v$  depends on the three angles between each pair of plane:

$$w_v = w_{\text{ang}}(\alpha_1, \alpha_2, \alpha_3) = A + (1 - A) \exp\left(-\frac{\sum_{i \in \{1,2,3\}} (\alpha_i - \pi/2)^2}{2\rho^2}\right) \quad (16)$$

The corner count terms correspond to potentials of order up to 8 in the context of MRFs